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Arbitrage, Cointegration, and Testing the Unbiasedness Hypothesis in Financial Markets

Robin J. Brenner and Kenneth F. Kroner*

Abstract

We use a no-arbitrage, cost-of-carry asset pricing model to show that the existence of cointegration between spot and forward (futures) prices depends on the time-series properties of the cost-of-carry. We argue that the conditions for cointegration are more likely to hold in currency markets than in commodity markets, explaining many of the empirical results in the literature. We also use this model to demonstrate why the forward rate forecast error, the basis, and the forward premium are serially correlated, and to develop econometric tests of the “unbiasedness hypothesis” (sometimes called the “simple efficiency hypothesis”) in various financial markets. The unbiasedness hypothesis is so prevalent in the finance literature that many tests for it have been developed. We examine four of the common tests and use our cointegration results to demonstrate why each of these tests *should* reject the null hypothesis of unbiasedness. We find strong support for our hypothesis in the existing empirical literature.

I. Introduction

Stochastic trends are prevalent in financial data.¹ For example, stock prices, foreign exchange rates, forward prices, and futures prices are known to have stochastic trends. A crucial implication of this in empirical finance is that the set of statistical models and tests that can be used to test financial theory are restricted, because many popular models and tests are inappropriate in the presence of stochastic trends. Recently, much attention has been given to the possibility that two or more assets might share the same stochastic trend; i.e., that the assets

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¹A variable y_t has a stochastic trend if its first difference, $y_t - y_{t-1}$, has a stationary invertible ARMA representation plus a deterministic component. For example, random walks have stochastic trends because their first difference is white noise.

might be cointegrated. For example, the following sets of financial time series (among many others) have been found to be cointegrated: sets of exchange rates (Baillie and Bollerslev (1989)); foreign currency spot and futures rates (Kroner and Sultan (1993)); interest rates of different maturities (Engle and Granger (1987)); interest rates in different countries (Akella and Patel (1991)); foreign currency spot and forward rates (Barnhart and Szakmary (1991)); dividends and prices (Campbell and Shiller (1987)); equity markets in different countries (Taylor and Tonks (1989)); stock prices within a given industry (Cerchi and Havenner (1988)); and size-ranked portfolios (Bossaerts (1988)). Interestingly, other sets have been found not to be cointegrated, such as commodity spot and futures prices (Baillie and Myers (1991)) and purchasing power parity relationships (Corbae and Ouliaris (1988)).

Cointegration is important because, as shown in Engle and Granger (1987), the presence of common stochastic trends further restricts the set of statistical models that can be used to test and implement financial theories. In particular, error correction models, which can be interpreted as models in which this period's price change depends on how far the system was out of long-run equilibrium last period, become necessary. The traditional solution of first differencing the data imposes too many unit roots in the system, invalidating standard inference procedures.² These problems become particularly important in finance when testing for market efficiency, when determining empirical hedge ratios, when constructing forecasting models, or when implementing many other financial models using multivariate time-series data.

While there exists a rapidly growing empirical literature on cointegration in financial markets, comparatively little has been done to examine the theoretical reasons for cointegration in financial markets. One notable exception is Campbell and Shiller (1987),³ who show that if the discount rate is constant, then prices derived from present value models must share a stochastic trend with the income stream whose present value determines the price. For example, stock prices and dividends must be cointegrated; however, their results require the assumption of "controversial present value models for stocks and bonds" (Campbell and Shiller (1987), p. 1086). In this paper, we permit interest rates to be stochastic and use only widely accepted no-arbitrage arguments to explain why some markets, such as currency spot and forward (futures) markets, are cointegrated, while other markets, such as commodity spot and forward (futures) markets, are not cointegrated. We also discuss the implications of our results on some of the existing tests for the unbiasedness hypothesis in financial markets. Because of the importance of the unbiasedness hypothesis in financial theory, many tests for it have been developed. We examine four of these tests and use our cointegration results to demonstrate why these tests reject unbiasedness. Furthermore, our model explains why shocks to the basis and forward premium are persistent and why strong serial correlation exists in the forward forecast error.

The structure of this paper is as follows. Section II derives the conditions required for cointegration to exist between spot and forward (futures) markets.

²For an interesting demonstration of this, see Barnhart and Szakmary's (1991) examination of the unbiasedness hypothesis in foreign exchange markets.

³Other exceptions include Bossaerts (1988) and Dwyer and Wallace (1992).

We argue that these conditions are likely to hold in currency markets, but not in commodity markets, explaining many of the empirical results in the literature. We also show that the optimal hedging and forecasting models are market-specific. Section III discusses the implications of our results on some existing tests for the unbiasedness hypothesis and shows why many of the existing tests reject unbiasedness. Empirical results from the existing literature strongly support our conclusions. We also demonstrate that the unbiasedness hypothesis cannot hold in commodity markets. Section IV concludes.

II. Cointegration in Asset Markets

Previewing our results in this section of the paper, we show that the forward (futures) price of an asset is

$$\text{Ln } S_t - \text{Ln } f_{t|t-k} = c - \text{Ln } D_{t|t-k} + \nu_t,$$

where S_t is the spot price at time t , $f_{t|t-k}$ is the value of a forward (futures) contract at time $t-k$, which expires at time t , and $D_{t|t-k}$ is the expected net cost-of-carry, or “differential,” over the life of the forward (futures) contract. Therefore, the results of tests for cointegration between spot and forward (futures) prices depend entirely on the time-series properties of the differential. If the differential has a stochastic trend, then spot and forward (futures) prices will tend to drift apart, and they would not be cointegrated. On the other hand, if the differential is stationary, then spot and forward (futures) prices are tied together, and they would be cointegrated.

To relate our results to the way cointegration tests and unbiasedness hypothesis tests are conducted in the literature, it is necessary to develop these results in the framework of an asset-pricing model for futures and forward contracts. To accomplish this task with mathematical rigor while retaining clear financial intuition, we assume a standard and well-known price process and use only standard no-arbitrage arguments to develop our pricing model. Specifically, we assume that the continuously compounded returns from the spot asset are normally distributed with mean μ and variance $\sum_{i=1}^n \gamma_i^2$. Mathematically, the precise assumption is that the spot price, S_t , evolves according to the n -factor geometric Brownian motion,

$$(1) \quad dS_t = \mu S_t dt + \sum_{i=1}^n \gamma_i S_t dW_{i,t},$$

where $W_{i,t}$, $i = 1, \dots, n$, are n independent standard Brownian motions that represent n independent sources of uncertainty (or n “factors”), μ is the instantaneous expected return, and γ_i are the diffusion coefficients. Letting $n = 1$ gives the popular diffusion process used in the seminal option pricing paper of Black and Scholes (1973),

$$dS_t = \mu S_t dt + \gamma_1 S_t dW_{1,t}.$$

In this special case, μ and γ_1 are the mean and standard deviation of the continuously compounded returns, respectively.

The spot process given in (1) can be viewed as a continuous-time random walk. To see this, solve equation (1) and take natural logarithms. This gives

$$(2) \quad \text{Ln } S_t - \text{Ln } S_{t-k} = \left(\mu - \frac{1}{2} \sum_{i=1}^n \gamma_i^2 \right) k + \sum_{i=1}^n \gamma_i [W_{i,t} - W_{i,t-k}].$$

This says that the change in the natural log of the spot price is equal to a constant plus an independent and identically distributed $N(0, k \sum_{i=1}^n \gamma_i^2)$ residual. Therefore, (1) implies that the natural log of the spot price has a stochastic trend. Of course, we could have obtained the same result much more directly by assuming that $\Delta \text{Ln } S_t$ follows an ARMA process, but we use the continuous-time specification here in order to permit a closed-form solution for futures prices later in this paper. First, however, we consider the pricing of forward contracts.

Forward contracts can usually be priced using a no-arbitrage pricing relationship between the forward and spot prices.⁴ Consider first the case of forward contracts on foreign exchange. Investors will be indifferent between i) investing in domestic bonds, and ii) converting domestic funds into foreign-denominated funds at the spot rate, investing in foreign bonds, and converting these funds back into domestic funds at the previously contracted forward rate. This no-arbitrage rule, commonly known as covered interest rate parity, has a simple mathematical expression. Let S_t be the domestic value of a foreign currency at time t , $f_{t|t-k}$ be the domestic value of a forward contract at time $t-k$ that expires at time t , and $P_{t|t-k}^d$ ($P_{t|t-k}^f$) be the price of a domestic (foreign) pure discount bond at time $t-k$ that pays one dollar at time t . Notice that $P_{t|t-k}^d = e^{-kr_{t|t-k}^d}$ and $P_{t|t-k}^f = e^{-kr_{t|t-k}^f}$, where $r_{t|t-k}^d$ ($r_{t|t-k}^f$) is the domestic (foreign) k -period interest rate at time $t-k$. Then the no-arbitrage pricing rule is

$$(3) \quad f_{t|t-k} = S_{t-k} \frac{P_{t|t-k}^f}{P_{t|t-k}^d} = S_{t-k} \cdot D_{t|t-k},$$

where $D_{t|t-k} = P_{t|t-k}^f / P_{t|t-k}^d$ is the cost-of-carry, or “differential.” Intuitively, equation (3) says that today’s forward price is equal to today’s spot price, adjusted by the difference between domestic and foreign interest rates.

Consider next the case for storable commodity forward prices. In this case, the investor would be indifferent between i) buying the commodity at the spot price and holding it, thereby incurring the storage costs and receiving the convenience yields, and ii) investing in the risk-free bond and buying the commodity later at the previously contracted forward price. Therefore, the pricing model for commodity forward contracts is similar to the model for currency forward contracts, except

⁴Throughout this paper, we make the common frictionless market and short-sale assumptions that allow the implementation of arbitrage-based trading strategies to enforce the cost-of-carry relationship. Such assumptions, though appropriate for (for instance) financial futures markets, are often inappropriate in commodity markets when the spot asset is in low supply. At such times, it becomes difficult to implement a strategy requiring short sales of the spot asset, and the forward price can, therefore, fall far below the cost-of-carry price. The reader should keep this in mind as an alternative explanation for the empirical commodity market results cited in this paper. However, verifying that such supply shortages cause the commodity market results we cite could be difficult. For a more detailed discussion of this topic, see Chapter 4 of Duffie (1989).

that the foreign interest rate is replaced by storage costs (including convenience yields). More rigorously, equation (3) holds, with $P_{t|t-k}^f = e^{kC_{t|t-k}}$, where $C_{t|t-k}$ is the storage costs (including convenience yield) from $t - k$ until t , expressed as a yield.

Finally, for forward prices on equities with known dividends, the investor would be indifferent between i) purchasing the stock and collecting the dividends, and ii) investing in a risk-free bond and buying the stock later at the previously contracted forward price. Therefore, the pricing relationship will be similar to the above except that the foreign interest rate is replaced with the continuous dividend yield on the stock between $t - k$ and t . So $P_{t|t-k}^f = e^{-kV_{t|t-k}}$, where $V_{t|t-k}$ is the dividend yield on the stock between time $t - k$ and time t .

Next, consider the pricing of futures contracts. Amin and Jarrow (1991) show that an equation similar to (3) holds for futures prices if we assume both that $P_{t|t-k}^f$ and $P_{t|t-k}^d$ fit into the Heath, Jarrow, and Morton (1992) framework, and that S_t , $P_{t|t-k}^d$, and $P_{t|t-k}^f$ are all correlated. Defining $F_{t|t-k}$ to be the price at time $t - k$ of a futures contract that expires at time t , a closed form solution for the futures price exists under these assumptions and is given by

$$(4) \quad F_{t|t-k} = S_{t-k} \frac{P_{t|t-k}^f}{P_{t|t-k}^d} \exp \{Q_{t|t-k}\} = S_{t-k} \cdot D_{t|t-k} \exp \{Q_{t|t-k}\},$$

where $Q_{t|t-k}$ is an adjustment term for the marking-to-market feature of futures contracts. This adjustment term depends on the volatilities of the interest rate and spot processes, and decreases to zero as $k \rightarrow 0$. See Amin and Jarrow (1991) both for a more rigorous discussion of the assumptions necessary to derive this result and for the mathematical formula for $Q_{t|t-k}$.

Taking the natural logarithm of equations (3) and (4) gives the linear relationship between the logarithms of the spot price, forward (futures) price, and differential,

$$(5a) \quad \text{Ln } f_{t|t-k} = \text{Ln } S_{t-k} + \text{Ln } D_{t|t-k},$$

$$(5b) \quad \text{Ln } F_{t|t-k} = \text{Ln } S_{t-k} + \text{Ln } D_{t|t-k} + Q_{t|t-k}.$$

Two implications of (5) are immediately apparent. First, if the logarithms of the spot price and the differential are not cointegrated with cointegrating vector $(1, -1)$, then (5) implies that the forward (futures) price has a stochastic trend. A sufficient condition for this is that $\text{Ln } D_{t|t-1}$ is stationary. Second, the forward premium (or basis), $\text{Ln } S_{t-k} - \text{Ln } f_{t|t-k}$ (or $\text{Ln } S_{t-k} - \text{Ln } F_{t|t-k}$), is serially correlated if the logarithm of the cost-of-carry is serially correlated. Therefore, the persistence of shocks to the forward premium (basis) will be the same as the persistence of shocks to the cost-of-carry. This can explain the findings of Backus and Gregory (1989) and others, who find evidence of strong persistence in the forward premium.

To obtain the relationship between a forward (futures) rate and the realized spot rate, substitute equation (5) into equation (2). This gives

$$\text{Ln } S_t - \text{Ln } f_{t|t-k} = \left(\mu - \frac{1}{2} \sum_{i=1}^n \gamma_i^2 \right) k - \text{Ln } D_{t|t-k}$$

$$(6a) \quad + \sum_{i=1}^n \gamma_i [W_{i,t} - W_{i,t-k}],$$

for forward contracts and

$$(6b) \quad \begin{aligned} \text{Ln } S_t - \text{Ln } F_{t|t-k} &= \left(\mu - \frac{1}{2} \sum_{i=1}^n \gamma_i^2 + Q_{t|t-k} \right) k - \text{Ln } D_{t|t-k} \\ &+ \sum_{i=1}^n \gamma_i [W_{i,t} - W_{i,t-k}], \end{aligned}$$

for futures contracts. The only difference between the relationship for forward prices and the relationship for futures prices is the marking-to-market adjustment term, $Q_{t|t-k}$, which appears in the constant in the futures equation. We placed this term into the constant because it is nonstochastic. Equation (6) says that there exists a linear relationship between the natural logarithms of the forward (futures) rate and the realized spot rate. Their difference is composed of three terms. The first term is an expected change term that may be either positive or negative depending on the magnitudes of the parameters involved. The second term is the differential over the remaining life of the forward (futures) contract, and is the key to the results that follow. This term can contain the effects of domestic and foreign interest rates, storage costs, convenience yields, and dividend yields, depending on the market under consideration. The third term is a random noise term due to the realization of the underlying economic factors, W_i , and is independent and identically distributed $N(0, k \sum_{i=1}^n \gamma_i^2)$.

We are now prepared to state the following proposition.

Proposition. Assume the spot price follows the diffusion model (1). For forward prices, assume the no-arbitrage assumption (3) holds and, for futures prices, assume $P_{t|t-k}^d$ and $P_{t|t-k}^f$ satisfy the correlation conditions given in Amin and Jarrow (1991). Then:

- i) If the differential does not have a stochastic trend, the natural logs of the spot and forward (futures) prices at any lead or lag must be cointegrated, with cointegrating vector $(1, -1)$.
- ii) If the differential has a stochastic trend but is not cointegrated with the natural log of the spot price with cointegrating vector $(1, -1)$, then any leads or lags of the natural logs of the spot price, the forward (futures) price, and the differential will form a trivariate cointegrated system, with cointegrating vector $(1, -1, 1)$.

Proof. See the Appendix.

The intuition for how linear no-arbitrage pricing formulas lead to cointegrated systems is straightforward. Arbitrage-based pricing duplicates one asset with a combination of other assets. So if the original asset has a stochastic trend, then the duplicated asset should have the same stochastic trend. But because variables are cointegrated if and only if they share a stochastic trend, the relationship between arbitrage pricing and cointegration should not be surprising. Also, as linear arbitrage pricing formulas give the exact combination of other assets needed to

duplicate one asset, it should not be surprising that such linear formulas not only lead to cointegrated systems of assets, but also give the exact combinations of the assets needed to establish cointegration.

One implication of this proposition is that if the differential has a stochastic trend, then spot and forward (futures) prices will *not* be cointegrated by themselves; the differential must be included in the system to find cointegration. This result contradicts a large body of literature that argues that spot and forward prices should be cointegrated,⁵ but the intuition is clear. The no-arbitrage assumption implies an equilibrium-type relationship between the forward premium (basis) and the differential. So, if the differential is stationary, the spot price and the forward (futures) price can never drift far apart. But if the differential has a stochastic trend, then the forward premium (basis) would also have a stochastic trend, implying that the spot and forward (futures) prices could drift far apart, and they would not be cointegrated.

Two important observations merit mention. First, many studies in the literature examine cointegration between current forward (futures) and realized spot prices (i.e., between $\ln S_t$ and $\ln f_{t|t-k}$), while many others examine cointegration between contemporaneous forward (futures) and spot prices (i.e., between $\ln S_{t-k}$ and $\ln f_{t|t-k}$). Our proposition provides the conditions under which cointegration exists at any lead or lag of the spot and forward (futures) prices. So, for example, if the differential is stationary, then the forward (futures) and the realized spot will be cointegrated, as will the forward (futures) and the contemporaneous spot.

Second, it is important to recognize that this proposition assumes that the time to expiration of the forward (futures) contract, k , is fixed, while the time of expiration, t , is changing. Many researchers, however, examine the relationship between the spot and futures prices as the time to expiration k decreases to zero, holding the expiration date t fixed. From equation (5b), we see that any regression of $\ln S_{t-k}$ on $\ln F_{t|t-k}$ for a fixed expiration date t has a residual that converges to zero as $k \rightarrow 0$, no matter what the time-series properties of the differential are. In other words, the spot and futures prices cannot drift apart if t is fixed. This means that the variance of this residual is changing through time (it is converging to zero), implying that the residual is not covariance stationary. Therefore, cointegration cannot exist.⁶ Similarly, instead of allowing a futures contract to expire, many other researchers roll over to the next nearest contract when expiration approaches (for example, within two weeks). Theoretically, the variance of the residual from this regression is still time-varying, meaning that cointegration still cannot theoretically hold. In practice, however, the empirical tests are unlikely to pick this up because the cointegration tests usually applied have very little power to detect shrinking variances. Put differently, cointegration can only exist when studying time series of futures prices with fixed and constant time to maturity. This is an important

⁵See, for examples, Campbell and Shiller (1987), Hakkio and Rush (1989), and Lai and Lai (1991). On the other hand, Baillie and Myers (1991) and Dwyer and Wallace (1992) correctly recognize that spots and forwards need not be cointegrated.

⁶On the other hand, if one modified the strict definition of cointegration to permit time-varying variances in the cointegrating regression residuals, then S_{t-k} and $F_{t|t-k}$ would be cointegrated as $k \rightarrow 0$.

result, because most empirical studies using futures prices use time series in which each observation has a different time to maturity.

Because of the central role of the differential in the above proposition, it will be informative to examine the stochastic properties of the differential in different kinds of markets. Consider first the market for commodities, where the log-differential can be written as $\text{Ln } D_{t|t-k} = \text{Ln}(e^{kC_{t|t-k}}) - \text{Ln } P_{t|t-k}^d = k(r_{t|t-k}^d + C_{t|t-k})$. Assuming that storage costs and convenience yields are not stochastic, the stochastic properties of $\text{Ln } D_{t|t-k}$ will be identical to the stochastic properties of $r_{t|t-k}^d$. Then, if interest rates have a stochastic trend, the differential will also have a stochastic trend. Whether or not interest rates have a stochastic trend is open to debate. There is substantial evidence that interest rates do have a stochastic trend, but there is also substantial evidence that they do not. See, for example, Bradley and Lumpkin (1992), Campbell and Shiller (1987), Engle and Granger (1987), Gonzalo and Granger (1991), Hall, Anderson, and Granger (1992), or Shea (1992), among many others, for evidence favoring a stochastic trend. Alternatively, see Fama and Bliss (1987), Sanders and Unal (1988), or Chan, Karolyi, Longstaff, and Sanders (1992), among many others, for evidence to the contrary. Along these lines, Brenner and Kroner (1993) demonstrate that some of the popular existing theoretical interest rate models imply that interest rates have a stochastic trend, while others do not.

If interest rates have a stochastic trend, then spot and forward (futures) prices will *not* be cointegrated by themselves, and the differential should be included in the cointegrating vector. This same result holds if convenience yields are stochastically trended, as long as interest rates and convenience yields are not cointegrated with cointegrating vector $(1, -1)$. Table 1 provides a sample of recent empirical studies on cointegration in commodity markets. The conclusion from this table is that spots and futures prices for commodities are clearly not cointegrated, providing support for our proposition. We should, therefore, look for cointegration either among the spot, futures, and interest rates, or among the spot, futures, interest rates, and the noninterest rate random elements of the cost-of-carry. To the best of our knowledge, this has not yet been done in the literature.

Next, consider the currency market, where $\text{Ln } D_{t|t-k} = k(r_{t|t-k}^d - r_{t|t-k}^f)$ will likely not have a stochastic trend. This is because the same sets of underlying economic forces likely drive interest rates in both countries, so differencing the domestic and foreign interest rates will eliminate the trend. Notice that we are essentially arguing that if interest rates in different countries have stochastic trends, they are likely to be cointegrated. See Akella and Patel (1991) for mixed empirical support for this argument. Alternatively, the differential will also be stationary if both sets of interest rates are stationary (whether or not they share underlying economic forces). In either case, our proposition says that we would expect the natural logarithms of the spot and forward (futures) prices to be cointegrated with cointegrating vector $(1, -1)$. This contrasts with the results above, where we argued that spot and forward (futures) prices for commodities are likely not to be cointegrated. See Table 2, which provides a brief summary of some of the recent empirical evidence on this issue in currency markets. Clearly, our theoretical results are strongly supported by the empirical evidence because cointegration is

TABLE 1
Summary of Cointegration Results for Commodity Markets

Study	Asset	Sample	Frequency	Contract	β	CI
Baillie and Myers (1991)	Beef	1981–1982	daily	F	–0.970	N
	Coffee			F	–0.926	N
	Corn			F	–1.058	N
	Cotton			F	–0.624	N
	Gold			F	–1.337	N
	Soybeans			F	–0.280	N
Bessler and Covey (1991)	Cattle	1985–1986	daily	F	–1.040	N
Chowdhury (1991)	Copper	74.7– 88.6	quarterly	F	–2.313	N
	Lead			F	–1.965	N
	Tin			F	–11.305	N
	Zinc			F	–0.788	N
Schroeder and Goodwin (1991)	Live hogs	1975–1989	daily	F	*	N

This table summarizes the cointegrating vectors estimated in various studies of commodity futures markets. The column labeled "Contract" gives an F if forward prices were used in the study and an f if futures prices were used; the column labeled β gives the estimated second element of the cointegrating vector, so that $(1, \beta)$ is the estimated cointegrating vector; and the column labeled "CI" gives a Y if cointegration was found, and an N if it was rejected.

*Schroeder and Goodwin (1991) ran a separate test for cointegration for each year in their data set, and found no cointegration for virtually every year. They did not report β .

invariably found, and with only a few exceptions, the cointegrating vector in these studies ranges between $(1, -0.95)$ and $(1, -1.03)$.

Finally, consider the market for equities, where the logarithm of the differential is $\text{Ln } D_{t|t-k} = k(r_{t|t-k}^d - V_{t|t-k})$. The stock price and its forward (futures) price will be cointegrated if $\text{Ln } D_{t|t-k}$ is stationary or, equivalently, if the spread between the interest rate and the dividend yield, $r_{t|t-k}^d - V_{t|t-k}$, is stationary. This would occur if the interest rates and dividend yields are cointegrated or if they are both stationary. Otherwise, the appropriate cointegrating relationship would involve the stock price, the forward (futures) price, and the difference between the interest rate and the dividend yield. To the best of our knowledge, the question of whether equity spot and forward (futures) prices are cointegrated has not yet been addressed in the literature.

The implications of this proposition are far reaching. For example, it should impact the way we construct forecasting models, hedging models, or tests of the unbiasedness hypothesis in financial markets. The Granger Representation Theorem (Engle and Granger (1987)) says that when modeling cointegrated variables, an error correction term (ECT) should be included as a regressor. Also, Granger (1986) establishes that when forecasting cointegrated variables, the inclusion of an ECT as a regressor often results in better long-run forecasts than traditional time-series models. For currencies, the ECT is the forward premium (basis), while for commodities, it is the forward premium (basis) minus the sum of the interest rate and convenience yield. It is important to recognize that because of the different results on cointegration in these two markets, we obtain different forecasting models, hedging models, and tests for the unbiasedness hypothesis in these two markets.

TABLE 2
Summary of Cointegrating Vectors for Foreign Exchange Markets

Study	Asset	Sample	Frequency	Contract	β	CI
Frenkel (1981)	UK	73.06–79.07	monthly	f	–0.956	?
	Germany			f	–0.971	?
	France			f	–0.843	?
Longworth (1981)	Canada	70.07–78.12	monthly	f	–1.023	?
		71.01–78.12		f	–1.023	?
		71.01–72.12		f	–0.922	?
		73.01–74.12		f	–0.877	?
		75.01–76.12		f	–0.729	?
		77.01–78.12		f	–0.960	?
		71.01–76.10		f	–0.923	?
Baillie, Lippens and McMahon (1983)	UK	73.06–80.04	four weeks	f	–0.956	?
	Germany			f	–0.968	?
	France			f	–0.884	?
Edwards (1983)	UK	73.07–79.09	monthly	f	–0.953	?
	Germany	73.07–79.09		f	–0.971	?
	France	73.07–78.12		f	–0.830	?
	Italy	73.07–78.12		f	–0.949	?
Baillie and Bollerslev (1989)	UK	80.03–85.01	daily	f	–1.011	Y
	Germany			f	–0.980	Y
	France			f	–0.985	Y
	Italy			f	–0.989	Y
	Switzerland			f	–0.976	Y
	Japan			f	–0.848	Y
	Canada			f	–0.960	Y
Hakkio and Rush (1989)	UK	75.07–86.10	monthly	f	–0.990	Y
	Germany			f	–0.970	Y
Barnhart and Szakmary (1991)	UK	74.01–88.11	monthly	f	–0.998	Y
	Germany			f	–0.975	Y
	Japan			f	–0.991	Y
	Canada			f	–0.987	Y
Copeland (1991)	UK	1976–1990	daily	f	–0.999	Y
	Germany			f	–0.998	Y
	France			f	–0.996	Y
	Japan			f	–0.997	Y
	Switzerland			f	–1.001	Y
Lai and Lai (1991)	UK	73.07–89.12	monthly	f	–1.002	Y
	Germany			f	–0.998	Y
	Switzerland			f	–0.997	Y
	Japan			f	–0.994	Y
	Canada			f	–1.001	Y
Kroner and Sultan (1993)	UK	85.02–90.02	weekly	F	–0.993	Y
	Germany			F	–1.000	Y
	Switzerland			F	–1.000	Y
	Japan			F	–0.994	Y
	Canada			F	–1.010	Y

This table summarizes some cointegrating vectors estimated in studies of currency markets. The column labeled "Contract" gives an f if forward prices were used and an F if futures prices were used; the column labeled β gives the second element of the estimated cointegrating vector, so that $(1, \beta)$ is the estimated cointegrating vector; and the column labeled "CI" gives a Y if cointegration was found, an N if it was rejected, and a ? if it was not tested for.

To the best of our knowledge, this has not been recognized in the literature. We now turn to a close examination of other implications of this result on tests for unbiasedness in financial markets.

III. Implications of Our Results on Unbiasedness Hypothesis Tests

The joint assumptions of risk neutrality (or no-risk premium) and rationality (so that speculators cannot expect to make excess returns) are so central in many finance models that their importance cannot be understated. Together, these two assumptions, which are usually derived from an uncovered interest rate parity assumption, have been called by some “simple efficiency” (Hansen and Hodrick (1980)), “speculative efficiency” (Bilson (1981)), “the efficient markets hypothesis” (Baillie, Lippens, and McMahon (1983)), “market efficiency” (Hakkio and Rush (1989)), and “the unbiasedness hypothesis” (Hodrick and Srivastava (1984)).⁷ In order to comment on work done by other authors, we will, for expositional purposes, use the phrase “unbiasedness hypothesis” to refer to the joint hypothesis of risk neutrality and rationality.

Because the unbiasedness hypothesis is central to much of finance, a voluminous literature has developed that tests for its relevance. See, for example, the surveys by Boothe and Longworth (1986) or Hodrick (1987). One test that is becoming popular argues that if the spot and forward prices have a stochastic trend, then a necessary condition for the unbiasedness hypothesis to hold is cointegration between the spot and forward prices with cointegrating vector $(1, -1)$. As a result, cointegration tests are often used to test for unbiasedness (see, for example, Hakkio and Rush (1989), Lai and Lai (1991), Shen and Wang (1990), and Sephton and Larsen (1991), among others). A second test involves running the regression,⁸

$$(7) \quad \text{Ln } S_{t+k} = \alpha + \beta \text{Ln } f_{t+k|t} + \epsilon_t,$$

and testing if $(\alpha, \beta) = (0, 1)$ and ϵ_t is serially uncorrelated. Ignoring risk, the intuition is that if all relevant information is immediately impounded into asset prices, then, on average, the forward rate should equal the realized spot rate, and there should be no information left in the residuals to help predict future spot rates. This test is applied by Frenkel (1981), Longworth (1981), and Edwards (1983), among others.

Several other popular methods of testing for the unbiasedness hypothesis are based on equation (7). For example, setting $k = 1$ and differencing (7) gives the regression model,

$$(8) \quad \Delta \text{Ln } S_{t+1} = \alpha + \beta \Delta \text{Ln } f_{t+1|t} + \xi_t.$$

⁷This kind of efficiency differs from Fama’s informational efficiency, which is usually taken to mean that today’s price “fully reflects” all available information (Fama (1970)). Fama’s definition is more closely related to rationality, or to the second part of this joint hypothesis. These two hypotheses are difficult to separate. See Hsieh (1984).

⁸This regression is sometimes run in levels instead of logs, but in currency markets, natural logarithms are usually taken to overcome the Siegel paradox. The same comment holds for equations (8) and (9).

Again, the test for the unbiasedness hypothesis is a test of $(\alpha, \beta) = (0, 1)$ and uncorrelated residuals. Hakkio (1981) and Baillie, Lippens, and McMahon (1983), among others, run tests similar to this using differenced data. A fourth popular test is obtained by subtracting $\text{Ln } S_t$ from both sides of equation (7), giving

$$(9) \quad \Delta \text{Ln } S_{t+1} = \alpha + \beta(\text{Ln } f_{t+1|t} - \text{Ln } S_t) + \epsilon_t.$$

Again, the test is $(\alpha, \beta) = (0, 1)$ and ϵ_t is serially uncorrelated. This model is estimated by Bilson (1981), Hodrick and Srivastava (1986), and Baillie and McMahon (1989), among many others. See Baillie (1989) or Barnhart and Szakmary (1991) for further discussion of some of these models.

These four tests tend to give mixed results, making it difficult to conclude whether or not markets are unbiased. In general, though, the cointegration-based tests conclude that commodity markets are biased, while currency markets are unbiased. Also, the literature seems to indicate that regression (7) finds evidence favoring the unbiasedness hypothesis, while regressions (8) and (9) do not. In fact, empirical estimates of equation (9) often find that $\hat{\beta}$ is closer to zero than to one (see Fama (1984), for example, where all nine currencies gave a $\hat{\beta}$ less than zero), implying strong rejections of the unbiasedness hypothesis. We now take another look at these empirical tests for unbiasedness, and use our simple cost-of-carry model from Section II to give theoretical reasons for many of these empirical results.

First, consider the cointegration-based tests. Our proposition says that given our spot asset process, if $\text{Ln } D_{t|t-1}$ is stationary (as we expect in currency markets), then the spot and forward (futures) prices are cointegrated with cointegrating vector $(1, -1)$. On the other hand, if the differential has a stochastic trend (as might be expected in commodity markets, whether caused by stochastically trended interest rates or stochastically trended convenience yields), then the spot and forward (futures) prices cannot be cointegrated, and the unbiasedness hypothesis cannot hold. Instead, the spot price, forward price, and differential are cointegrated with cointegrating vector $(1, -1, -1)$. This suggests that the concept of unbiasedness is much more relevant in currency markets than in commodity markets. It can also explain the rejections of the unbiasedness hypothesis in commodity markets using cointegration-based tests by MacDonald and Taylor (1988), Schroeder and Goodwin (1991), Chowdhury (1991), and Bessler and Covey (1991), among others. It also explains the cointegration-based support for the unbiasedness hypothesis found in currency markets by Barnhart and Szakmary (1991), Baillie and Bollerslev (1989), Nugent (1990), and others. In conclusion, the results of cointegration-based unbiasedness tests depend entirely on the stochastic properties of the differential, and need say nothing about the rationality or risk neutrality of market participants.

Second, consider the test for the unbiasedness hypothesis from regression (7). Rewriting equation (6a) and setting $n = 1$ gives

$$(10) \quad \text{Ln } S_t = \left(k\mu - k\frac{\gamma^2}{2} - E[\text{Ln } D_{t|t-k}] \right) + \text{Ln } f_{t|t-k} + (\gamma[W_t - W_{t-k}] - \eta_{t|t-k}),$$

where the differential has been decomposed into its (deterministic) expectation, $E[\text{Ln } D_{t|t-k}]$, plus its (stochastic) forecast error, $\eta_{t|t-k}$. A test of $\alpha = 0$ in equation (7) is essentially a test of $E[\text{Ln } D_{t|t-k}] = k\mu - k(\gamma^2/2)$, and there is no reason for this restriction to hold. This explains the common finding in the literature (see Lai and Lai (1991), for example) that tests of $\beta = 1$ are not rejected while tests of $(\alpha, \beta) = (0, 1)$ are strongly rejected. Also, the residual term in (10), $\eta_{t|t-k} + \gamma[W_t - W_{t-k}]$, is composed of two terms: the forecast error from the differential and a white noise component. There is no reason for this composite error term to be white noise unless the differential is deterministic (i.e., unless $\eta_{t|t-k} = 0$). This can explain another common finding in the literature that the forward rate forecast errors, which are the residuals from equation (7), are strongly serially correlated (see Hsieh and Kulatilaka (1982), Canarella and Pollard (1986), or Baillie and Bollerslev (1990)). Essentially, any dynamics in the differential are transferred to the residual. To conclude, then, we expect tests of the unbiasedness hypothesis using regression (7) to reject either on the basis of $\alpha \neq 0$ or on the basis of serial correlation in the residuals.

Third, estimating equation (7) with OLS results in parameter estimates that are not normally distributed because the presence of stochastic trends violates the standard OLS assumptions (Park and Phillips (1989)), meaning that standard inference procedures are inappropriate.⁹ It was in part because of this problem that several researchers (since the mid-1980s) took first differences of their data before conducting unbiasedness hypothesis tests. However, if we assume that the differential does not have a stochastic trend (e.g., currency markets) and that $k = 1$, then differencing equation (10) gives

$$\begin{aligned}
 (11) \quad \text{Ln } S_t - \text{Ln } S_{t-1} &= -E(\text{Ln } D_{t|t-1}) + E(\text{Ln } D_{t-1|t-2}) \\
 &\quad + \text{Ln } f_{t|t-1} - \text{Ln } f_{t-1|t-2} + (\eta_{t|t-1} + \gamma[W_t - W_{t-1}]) \\
 &\quad - (\eta_{t-1|t-2} + \gamma[W_{t-1} - W_{t-2}]) \\
 &= \left(\mu - \frac{\gamma^2}{2} - E[\text{Ln } D_{t|t-1}] \right) + (\text{Ln } f_{t|t-1} - \text{Ln } f_{t-1|t-2}) \\
 &\quad + (\text{Ln } f_{t-1|t-2} - \text{Ln } S_{t-1}) + (\eta_{t|t-1} + \gamma[W_t - W_{t-1}]) .
 \end{aligned}$$

The term $\text{Ln } f_{t-1|t-2} - \text{Ln } S_{t-1}$ is called the “error correction term.” Comparing equation (11) and regression (8) reveals that (8) is missing the error correction term and, therefore, suffers from an omitted regressor bias. It is straightforward to show that the covariance between the omitted regressor and the included variable is negative. Therefore, because the coefficient on the omitted regressor is positive, the estimated $\hat{\beta}$ from regression (8) is downward biased (i.e., $E(\hat{\beta}) < 1$). Not surprisingly, tests based on (8) tend to reject, with $\hat{\beta}$ usually less than one (see, for example, Hegde and McDonald (1986)).

Equation (11) reveals that an appropriate test for unbiasedness can be obtained from the regression,

$$(12) \quad \Delta \text{Ln } S_t = \alpha + \beta \Delta \text{Ln } f_{t|t-1} + \delta (\text{Ln } f_{t-1|t-2} - \text{Ln } S_{t-1}) + \epsilon_t .$$

⁹This problem was not widely recognized until the mid to late 1980s, so many empirical studies using equation (7) were making inferences based on incorrect distributions.

In this regression, the unbiasedness hypothesis is satisfied if $(\alpha, \beta, \delta) = (0, 1, 1)$ and the residuals are serially uncorrelated. The advantage of regression (12) over (8) as a test of the unbiasedness hypothesis is that in (12), standard hypothesis testing is appropriate, while it is not in (8). However, based on (11), we do not expect $\alpha = 0$ or ϵ_t to be serially uncorrelated. Perhaps the only application of (12) in the literature is Hakkio and Rush (1989), who add terms to (12) to capture the serial correlation in the residuals, and reject unbiasedness, in part, because these extra terms belong. Our results show that this is to be expected unless the differential is white noise.

Equations (11) and (12) are developed assuming the differential is stationary. However, if the differential has a stochastic trend (e.g., commodity markets), then the appropriate error correction model would be

$$\begin{aligned} \text{Ln } S_t - \text{Ln } S_{t-1} = & \left(\mu - \frac{\gamma^2}{2} \right) \\ & + (\text{Ln } f_{t|t-1} - \text{Ln } f_{t-1|t-2}) - (\text{Ln } D_{t|t-1} - \text{Ln } D_{t-1|t-2}) \\ & - (\text{Ln } S_{t-1} - \text{Ln } f_{t-1|t-2} - \text{Ln } D_{t-1|t-2}) + \gamma [W_t - W_{t-1}], \end{aligned}$$

and the regression comparable to (12) is

$$\begin{aligned} (13) \quad \Delta \text{Ln } S_t = & \alpha + \beta_f \Delta \text{Ln } f_{t|t-1} + \beta_d \Delta \text{Ln } D_{t|t-1} \\ & + \delta (\text{Ln } f_{t-1|t-2} - \text{Ln } S_{t-1} - \text{Ln } D_{t-1|t-2}) + \epsilon_t. \end{aligned}$$

In this regression, there are no restrictions on $(\alpha, \beta_f, \beta_d, \delta)$ that will support the unbiasedness hypothesis. This corroborates our earlier conclusion that the concept of unbiasedness might not be relevant in commodity markets. If the differential has a stochastic trend, then the unbiasedness hypothesis cannot hold. To the best of our knowledge, this has not yet been recognized in the literature. The intuition is that the difference between the realized spot and current forward (futures) price contains the differential, and if this has a stochastic trend, then it must be strongly serially correlated. But serial correlation in the forecast error violates the unbiasedness hypothesis. It is not surprising, then, that studies in commodity markets invariably reject unbiasedness (for examples, see Bessler and Covey (1991) or Chowdhury (1991)).

Fourth, we turn our attention to regression (9) as a test for the unbiasedness hypothesis. The Granger Representation Theorem in Engle and Granger (1987) shows that if variables are cointegrated, then their joint data-generating mechanism can be written as an error correction model. To demonstrate why regression (9) rejects the unbiasedness hypothesis, we apply the Granger Representation Theorem to the model in Section II above. Consider the situation where the differential does not have a stochastic trend (e.g., currency markets).¹⁰ The geometric diffusion model given in (1) and the no-arbitrage condition in (3) or (4) imply

$$\begin{aligned} (14) \quad \text{Ln } S_t &= \alpha_s + \text{Ln } S_{t-1} + \zeta_t, \\ \text{Ln } f_{t+1|t} &= \alpha_f + \text{Ln } S_t + \eta_{t+1|t}, \end{aligned}$$

¹⁰Because the unbiasedness hypothesis cannot hold if the differential has a stochastic trend, we only examine the case where the differential is stationary.

$$\begin{aligned}
\text{where } \zeta_t &= \sum_{i=1}^n \gamma_i [W_{i,t} - W_{i,t-1}], \\
\eta_{t+1|t} &= \text{Ln } D_{t+1|t} - E(\text{Ln } D_{t+1|t}), \\
\alpha_s &= \left(\mu - \frac{1}{2} \sum_{i=1}^n \gamma_i^2 \right), \\
\alpha_f &= \begin{cases} E(\text{Ln } D_{t+1|t}) & \text{for forward prices,} \\ E(\text{Ln } D_{t+1|t}) + Q_{t+1|t} & \text{for futures prices.} \end{cases}
\end{aligned}$$

Applying the Granger Representation Theorem to this system of equations yields¹¹

$$\begin{aligned}
(15) \quad \Delta \text{Ln } S_t &= \alpha_s + 0 \cdot z_{t-1} + \zeta_t, \\
\Delta \text{Ln } f_{t+1|t} &= \alpha_s - 1 \cdot z_{t-1} + \zeta_t + \eta_{t+1|t}, \\
\text{where } z_{t-1} &= -\alpha_f + \text{Ln } f_{t|t-1} - \text{Ln } S_{t-1}.
\end{aligned}$$

Rewriting this as a regression model gives

$$\begin{aligned}
(16) \quad \Delta \text{Ln } S_t &= a_s + \gamma_s z_{t-1} + \xi_{1t}, \\
\Delta \text{Ln } f_{t+1|t} &= a_d + \gamma_f z_{t-1} + \xi_{2t},
\end{aligned}$$

where (15) implies that $(\gamma_s, \gamma_f) = (0, -1)$ and ξ_{1t} is serially uncorrelated. Notice that the first equation of this system is the same as regression equation (9) if we place the constant part of the error correction term, $-\alpha_f$, into the equation's intercept, a_s . This regression has been used by several authors to test for the unbiasedness hypothesis (for examples, see Geweke and Feige (1979), Hansen and Hodrick (1980), Cumby and Obstfeld (1981), and Gregory and McCurdy (1984)). These authors test whether $\gamma_s = 1$, but equation (15) shows that we expect γ_s to be zero.¹² It is, therefore, not surprising that most researchers using this model find that γ_s is closer to zero than it is to one. For example, Fama (1984) finds that for all nine of the currencies on which he ran this regression, $\gamma_s = 1$ was rejected, but based on his reported standard errors, $\gamma_s = 0$ would have been rejected only *once*. Longworth (1981) finds $\hat{\gamma}_s = 0.05$, and Kroner and Sultan (1993) reject $\gamma_s = 1$ for all five currencies and reject $\gamma_s = 0$ only once. Furthermore, Kroner and Sultan (1993) never reject the hypothesis $\gamma_f = -1$, which is consistent with the theory presented here. See also Norrbin and Reffet (1993), who find strong evidence that $\gamma_s = 0$ and $\gamma_f < 0$ for all five currencies they examined. See Table 3 for a summary of some of the estimated γ_s s in the literature.

¹¹In this model, the same result could be obtained by subtracting $\text{Ln } f_{t|t-k}$ from both sides of (14), then substituting equation (2) into equation (14), setting $k = 1$, and rearranging terms.

¹²It is important to recognize that this conclusion rests on the assumption that the spot price dynamics are determined by equation (1) and the forward price is determined by covered interest parity. If instead, assumption (1) were placed on the forward price and the spot price were determined by covered interest parity, then γ' would be (1, 0). Based on the evidence cited below and conversations with international bankers, the assumption that forward prices are determined by covered interest parity, given a spot price, seems irrefutable.

TABLE 3
Summary of the γ_S Coefficient

Study	Asset	γ_S	$\gamma_S = 0$	$\gamma_S < 1$	Eff	Comments
Bilson (1981)	Canada	-0.804	*	*	Y	
	UK	0.628	*		Y	
	France	-0.849	*	*	Y	
	Germany	-0.208	*		Y	
	Italy	-0.372	*	*	Y	
	Switzerland	-0.184	*		Y	
	Japan	-0.665	*	*	Y	
Longworth (1981)	CD, 70.7-78.12	0.052	*		Y	
	CD, 71.1-78.12	-0.094	*		Y	
	CD, 71.1-72.12	-1.191	*		Y	
	CD, 73.1-74.12	-1.338	*		Y	
	CD, 75.1-76.12	-1.307	*		Y	
	CD, 77.1-78.12	0.509	*		Y	
	CD, 71.1-76.10	-0.947		*	Y	
Fama (1984)	Belgium	-1.580		*	N	
	Canada	-0.870	*	*	N	
	France	-0.870	*	*	N	
	Italy	-0.510	*	*	N	
	Japan	-0.290	*	*	N	
	Netherlands	-1.430	*	*	N	
	Switzerland	-1.140	*	*	N	
	UK	-0.900	*	*	N	
Boothe and Longworth (1986)	Germany	-1.320	*	*	N	
	Canada	-0.389	*	*	Y	
	France	-1.838		*	Y	
	Germany	-0.533	*		Y	
	Italy	-0.479	*	*	Y	
	Japan	0.480	*		Y	
	UK	-1.533	*	*	Y	
Backus and Gregory (1989)	Canada	0.561	*	*	N	
	France	-0.481	*		N	
	Japan	-0.503	*	*	N	
	UK	-2.243		*	N	
	Germany	-0.326	*	*	N	
Baillie (1989)	Germany	0.313		*	Y	other vars present
	Germany	0.208		*	Y	restricted model
Barnhart and Szakmary (1991)	UK	-1.650		*	Y	other vars present
	Germany	-3.630		*	Y	other vars present
	Japan	-0.586	*	*	Y	other vars present
	Canada	-2.020		*	Y	other vars present
Kroner and Sultan (1993)	Canada	-0.300	*	*	N	
	UK	0.260	*	*	N	
	Switzerland	-0.070	*	*	N	
	Japan	0.130	*	*	N	
	Germany	0.310		*	N	

This table summarizes results from estimating

$$\Delta \ln S_t = \alpha + \gamma_S (\ln S_{t-1} - \ln f_{\eta t-1}).$$

The column " $\gamma_S = 0$ " gives an asterisk if γ_S is insignificantly different from zero; the column " $\gamma_S < 1$ " gives an asterisk if γ_S is significantly less than 1; and the column labeled "Eff" gives a Y if the regression was run to test for the unbiasedness hypothesis, and an N otherwise.

One final implication is that these results can be used in Baillie's (1989) proposed test for unbiasedness. Baillie (1989) recommends a method of testing that uses restricted VARs in which one of the restrictions requires prior knowledge of the cointegrating vector. He recommends using either theory or prior empirical evidence to obtain the appropriate restriction, and the proposition in Section II can be used to obtain the correct restriction.

IV. Conclusion

The existing literature in empirical finance provides convincing evidence that certain financial assets are cointegrated while others are not. For example, it is widely accepted that foreign currency spot and futures prices are cointegrated with cointegrating vector $(1, -1)$, while commodity spot and futures prices are not cointegrated. This paper gives a theoretical foundation for many of these empirical findings. We show that if the net cost-of-carry is stationary, then spot and forward prices and spot and futures prices should be cointegrated with cointegrating vector $(1, -1)$, but if the differential has a stochastic trend, then these assets should not be cointegrated. We discuss the implications of these results on some popular existing tests for the unbiasedness hypothesis, and offer explanations for many of the empirical results in this literature.

We conclude that most of the existing tests for the unbiasedness hypothesis should be expected to result in rejections. This theoretical result, combined with the vast empirical literature that supports it, should cause us to question the common assumption of the unbiasedness hypothesis in financial models. It would seem that a no-arbitrage assumption such as covered interest rate parity would be a more reasonable assumption to use. Other results include a demonstration of why there is strong persistence in the forward premium and the basis. We argue that any persistence in the differential will manifest itself as persistence in the forward premium and the basis.

Appendix

Proof of Proposition.

i) Define $X'_t \equiv (\text{Ln } S_t, \text{Ln } f_{t|t-k})$. We have established in the text that if the differential does not have a stochastic trend, then both elements of X_t have a stochastic trend. Also, define $\alpha' \equiv (1, -1)$. Then, from equation (6a), $\alpha'X_t = c - d_t + v_t$, where d_t is stationary and v_t is white noise. So $\alpha'X_t$ is stationary. Because each element of X_t has a stochastic trend and $\alpha'X_t$ is stationary, the elements of X_t are cointegrated with cointegrating vector $\alpha' = (1, -1)$. Also, Engle and Granger (1987) show that if any variables are cointegrated, then they are cointegrated at any lead or lag as well. Therefore, any lead or lag of $\text{Ln } S_t$ and $\text{Ln } f_{t|t-k}$ will be cointegrated. This proof holds for futures prices if equation (6b) is used in place of equation (6a).

ii) Define $X'_t \equiv (\text{Ln } S_t, \text{Ln } f_{t|t-k}, \text{Ln } D_{t|t-k})$. We showed above that $\text{Ln } S_t$ has a stochastic trend, and that, if $\text{Ln } S_t$ and $\text{Ln } D_{t|t-k}$ are not cointegrated with cointegrating vector $(1, -1)$, then $\text{Ln } f_{t|t-k}$ also has a stochastic trend. So, if $\text{Ln } D_{t|t-k}$

has a stochastic trend, then each element of X_t has a stochastic trend. Define $\alpha' \equiv (1, -1, 1)$. Then by equation (6a), $\alpha'X_t = c + v_t$ with v_t white noise, so $\alpha'X_t$ is stationary. Because each element of X_t has a stochastic trend and $\alpha'X_t$ is stationary, the elements of X_t are cointegrated with cointegrating vector $\alpha' = (1, -1, 1)$. Finally, Engle and Granger (1987) show that, if variables are cointegrated, then they are cointegrated at any lead or lag as well. Therefore, any lead or lag of $\text{Ln } S_t$, $\text{Ln } f_{t|t-k}$ and $\text{Ln } D_{t|t-k}$ will be cointegrated. This proof holds for futures prices if equation (6b) is used in place of (6a). \square

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