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Time-Varying Distributions and Dynamic Hedging with Foreign Currency Futures

Kenneth F. Kroner and Jahangir Sultan*

Abstract

Most research on hedging has disregarded both the long-run cointegrating relationship between financial assets and the dynamic nature of the distributions of the assets. This study argues that neglecting these affects the hedging performance of the existing models and proposes an alternative model that accounts for both of them. Using a bivariate error correction model with a GARCH error structure, the risk-minimizing futures hedge ratios for several currencies are estimated. Both within-sample comparisons and out-of-sample comparisons reveal that the proposed model provides greater risk reduction than the conventional models. Furthermore, a dynamic hedging strategy is proposed in which the potential risk reduction is more than enough to offset the transactions costs for most investors.

I. Introduction

With the recent popularity of commodity and financial futures, the academic and financial communities have seen a renewed interest in hedging theories. In particular, several empirical investigations that search for the risk-minimizing investment portfolio have been conducted. This "hedging portfolio" has usually been found by regressing the returns to holding the spot asset on the returns to holding the hedging instruments. For example, let S_t be the price of a Japanese yen (in U.S. dollars) and F_t be the price of a yen futures contract. Then a U.S. investor who holds one dollar's worth of Japanese assets and wishes to minimize his exposure to exchange rate risk would go short in $\hat{\beta}$ dollars worth of futures contracts, where according to most empirical investigations $\hat{\beta}$ comes from the regression

$$(1) \quad S_t - S_{t-1} = \alpha + \beta (F_t - F_{t-1}) + \epsilon_t.$$

For examples, see Ederington (1979) and Hill and Schneeweis (1982), among many others.

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Two potential problems are prevalent in these empirical studies. First, if the spot and futures rates are cointegrated, then Regression (1) is misspecified because it involves overdifferencing the data and obscuring the long-run relationship between S_t and F_t (Engle and Granger (1987)). This implies a downward bias in $\hat{\beta}$ (Brenner and Kroner (1993)). Second, these studies implicitly assume that the risk in spot and futures markets is constant over time, implying that the minimum risk hedge ratio will be the same irrespective of when the hedging is undertaken. But this assumption contrasts sharply with reality because as new information is received by the market, the riskiness of each of these assets changes. See Bollerslev (1990) or Kroner and Sultan (1991) for evidence of this. This assumption implies that the risk-minimizing hedge ratio is time varying. Therefore, conventional models like (1) cannot produce risk-minimizing hedge ratios, raising important concerns regarding the risk reduction properties of conventional hedging models.

In this paper, we demonstrate a method of calculating the risk-minimizing futures hedge that addresses both of these issues, and apply the method to several different currencies. We propose and estimate a bivariate error correction model (ECM) in ΔS_t and ΔF_t with a GARCH error structure. The error correction term imposes the long-run relationship between S_t and F_t , and the GARCH error structure permits the second moments of the distribution to change through time. The time-varying hedge ratios can then be calculated from the estimated covariance matrix from the model. Both within-sample tests and out-of-sample tests reveal that this method leads to more effective hedges than the conventional method, suggesting that our proposed method is a potentially superior way to manage currency risk. We also propose a realistic hedging strategy that explicitly recognizes the transaction costs of rebalancing, and demonstrate the superiority of our proposed strategy over the conventional strategies. This study is organized as follows. Section II presents and compares the conventional hedging model and a time-varying hedge model. Section III discusses the data and Section IV offers empirical results. Section V concludes the study.

II. Hedging Theories

One of the simplest ways to hedge risk is using what we call the *naive* hedging strategy. This strategy suggests that to minimize exposure, an investor who is long in the spot market should sell a unit of futures today and buy the futures back when he sells the spot. Then, if the spot and futures prices both change by the same amount, the hedger's net position will be unchanged and he would have a perfect hedge.

But since spot and futures prices do not always move together, the regression in Equation (1) above was proposed, giving what we call the *conventional* hedging strategy. This strategy can be derived as follows. Let F_0 and F_1 be the purchase and the settlement prices, respectively, of a futures contract, and let S_0 and S_1 be the spot price at the time the futures contract is purchased and at the time it is settled, respectively. Also, suppose the investor has a fixed long position of one unit in the spot market and a short position of $-b$ units in the futures market. The random return to this portfolio, x , is

$$(2) \quad x = s - bf,$$

where s and f are changes in the spot and futures prices, respectively. Assume the investor faces the mean-variance expected utility function,

$$(3) \quad EU(x) = E(x) - \gamma \text{Var}(x),$$

where γ is the degree of risk aversion ($\gamma > 0$). The investor solves

$$(4) \quad \max_b EU(x) = \max_b \{E(s) + bE(f) - \gamma [\sigma_s^2 + b^2\sigma_f^2 - 2b\sigma_{sf}]\}$$

for b , giving the optimal number of futures contracts in the investor's portfolio,

$$(5) \quad b^* = \frac{E(f) + 2\gamma\sigma_{sf}}{2\gamma\sigma_f^2}.$$

If the futures rate follows a martingale (i.e., $E(F_1) = F_0$), Equation (5) reduces to¹

$$(6) \quad b^* = \frac{\sigma_{sf}}{\sigma_f^2}.$$

If the joint distribution of spot and futures is constant over time, one can extend this model to a multi-period hedging strategy by assuming a time-separable utility function. The solution for the sequence of hedge ratios $\{b_1, \dots, b_T\}$ then gives $b_i = b_j \forall i, j$, and the hedge ratio is often calculated as the least squares estimator from a time-series regression of ΔS_t on ΔF_t , as in Regression (1) above. Notice that with the assumptions made in developing this model, the optimal hedge ratio is equivalent to the risk-minimizing hedge ratio.

However, because the distribution of spot and futures prices is time varying, consider the following alternative model. Let f_t and s_t be the changes in the price of the futures and the spot between time t' and t , respectively, and define $-b_{t'}$ as the short position in futures at time t' . Then,

$$(7) \quad x_t = s_t - b_{t'}f_t \quad t' < t$$

is the payoff at time t to purchasing one unit of the spot and going short in $b_{t'}$ units of the futures at some time t' in the past. The investor chooses his optimal one-period holdings of futures at each time t by maximizing the expected utility function,

$$(8) \quad E_t U(x_{t+1}) = E_t(x_{t+1}) - \gamma\sigma_t^2(x_{t+1}),$$

where risk is now measured by *conditional* (not unconditional) variances, and the expectation and variance operators are subscripted with t to emphasize that they are

¹The martingale assumption implies that the expected returns from the portfolio are unaffected by the number of futures contracts held, so that risk minimization becomes an equivalent model. See McCurdy and Morgan (1988) for empirical evidence that weekly futures rates follow a martingale.

calculated conditional on information available at time t . The utility-maximizing hedge ratio at time t is

$$(9) \quad b_t^* = \frac{E_t(f_{t+1}) + 2\gamma\sigma_t(s_{t+1}, f_{t+1})}{2\gamma\sigma_t^2(f_{t+1})},$$

and assuming that futures prices are a martingale (i.e., $E_t(F_{t+1}) = F_t$), this simplifies to

$$(10) \quad b_t^* = \frac{\sigma_t(s_{t+1}, f_{t+1})}{\sigma_t^2(f_{t+1})}.$$

This is similar to the conventional hedge ratio except that time-varying conditional moments replace the time-invariant unconditional moments so the risk-minimizing hedge ratio will change through time as new information arrives to the market. This *conditional* model will reduce to the conventional model if the joint distribution of spot and futures is constant through time.

We propose a bivariate GARCH error correction model to estimate b_t^* . The error correction part of the model is necessary because spot and futures rates share the same long-run stochastic trend under weak assumptions, and the error correction term ensures that the long-run relationship between them is maintained. The GARCH part of the model permits the hedge ratio to change as new information arrives at the market, which should account for the empirical regularity of intertemporal instability in the hedging effectiveness of the conventional models (Grammatikos and Saunders (1983), Figlewski (1984)). A failure to account for this could prove costly to those who, for various reasons such as maturity mismatch, are required to update their hedge on a regular basis.

The bivariate GARCH error correction model requires parameterizing the first two conditional moments of the bivariate distribution of s_t and f_t . We model the first moment with a bivariate error correction model (Engle and Granger (1987)) and the second moment with a bivariate constant correlation GARCH(1,1) model (Bollerslev (1990); Kroner and Claessens (1991)). The econometric model we propose is

$$(11) \quad \begin{aligned} s_t &= \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \epsilon_{st} \\ f_t &= \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \epsilon_{ft}, \end{aligned}$$

$$(12) \quad \begin{bmatrix} \epsilon_{st} \\ \epsilon_{ft} \end{bmatrix} \Bigg| \Psi_{t-1} \sim N(0, H_t),$$

$$(13) \quad H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix},$$

$$(14) \quad \begin{aligned} h_{s,t}^2 &= c_s + a_s \epsilon_{s,t-1}^2 + b_s h_{s,t-1}^2 \\ h_{f,t}^2 &= c_f + a_f \epsilon_{f,t-1}^2 + b_f h_{f,t-1}^2, \end{aligned}$$

where Ψ_{t-1} is the information set at time $t - 1$. This model can be generalized to allow for moving average terms, lagged dependent variables, and other weakly exogenous variables in either the mean or the variance.

The term $(S_{t-1} - \delta F_{t-1})$ is the error correction term (ECT), which imposes the long-run restrictions into this short-run model. Engle and Granger (1987) show that if two variables are cointegrated, then their bivariate time series model should include an ECT. Brenner and Kroner (1993) show that spot and futures prices will be cointegrated if the interest rate differential is stationary. The intuition behind their argument is as follows. Covered interest rate parity gives an exact linear pricing relationship between the logarithms of spot prices, futures prices, domestic interest rates, and foreign interest rates.² In particular,

$$\text{Ln } F_t = \text{Ln } S_t + (\text{Ln } P_t^f - \text{Ln } P_t^d) + Y_t,$$

where P_t^f and P_t^d are one plus the foreign and one plus domestic interest rates, respectively, and Y_t is a nonstochastic adjustment term for the marking-to-market feature of futures contracts. Because cointegration is an econometric way of identifying linear combinations of variables that move together in the long run, the four variables ($\text{Ln } F_t$, $\text{Ln } S_t$, $\text{Ln } P_t^f$, and $\text{Ln } P_t^d$) should be cointegrated. Furthermore, if foreign and domestic interest rates move together in the long run, then the interest rate differential ($\text{Ln } P_t^f - \text{Ln } P_t^d$) will be stationary, meaning that $\text{Ln } S_t$ and $\text{Ln } F_t$ will move together in the long run, or share the same stochastic trend. In econometric terminology, $\text{Ln } S_t$ and $\text{Ln } F_t$ are said to be cointegrated with cointegrating vector $(1, -1)$. In contrast, Brenner and Kroner (1993) show that spots and futures prices for *commodities* will be cointegrated only if the interest rate itself is stationary, and there is substantial evidence that interest rates are nonstationary. So it is not surprising that we find cointegration in currency markets, while others fail to find cointegration in commodity markets (see also Baillie and Myers (1991)). The important implication is that hedging models in currency markets should incorporate an ECT, while hedging models in commodity markets need not incorporate an ECT.

A GARCH(1,1) model is used because of the substantial empirical evidence that this model adequately characterizes the dynamics in second moments of currency prices. For examples, see McCurdy and Morgan (1988) or Hsieh (1989) who find that the GARCH(1,1) model characterizes the second moments of currency prices better than high order ARCH models. Furthermore, McCurdy and Morgan (1988), among many others, also demonstrate that the ARCH-M model is inappropriate for currency markets, because there is no mean-variance tradeoff in these markets. And finally, because of the two-sided nature of currency markets, we would not expect asymmetries in the variance equations, such as those permitted by the EGARCH model. For a more thorough discussion of these issues, see the survey paper on ARCH modeling in finance by Bollerslev, Chou, and Kroner (1992). To summarize, then, based on results in the literature, we would expect our GARCH(1,1) model to perform at least as well as other GARCH models when constructing our hedge portfolios.

²This no arbitrage argument requires certain assumptions be placed on the domestic and foreign interest rate processes.

A number of other observations are worth mentioning here. First, the conventional model is nested within this GARCH(1,1) model. In particular, the conventional model can be tested against the conditional model by testing if $a_s = b_s = a_f = b_f = \alpha_{1s} = \alpha_{1f} = 0$. Second, the dynamic hedge ratio at time t can be computed as the ratio of conditional covariance between s and f to the conditional variance of f (both measured at time t), i.e., as

$$(15) \quad \hat{b}_t^* = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}$$

This gives us the time-varying hedge ratios that are based on conditional moments. Third, forecasts of the variances and covariances can be computed in much the same way as forecasts from vector ARMA models. This facilitates out-of-sample forecasts of the time-varying hedge ratios.

III. Data

Exchange rates used in this study are the natural logarithms of Thursday's spot and futures data from February 8, 1985, to February 23, 1990 (264 observations), for the British pound (BP), the Canadian dollar (CD), the German mark (DM), the Japanese yen (JY), and the Swiss franc (SF).³ This period coincides with the International Monetary Market (IMM) removing limit move restrictions on currency futures trading beginning February 2, 1985. The spot exchange rates are closing prices in New York, and the futures rates are IMM closing prices. Of the four futures contracts that are outstanding at any given time, we use the price of the nearest contract. To avoid thin markets and expiration effects, however, we roll over to the next nearest contract three weeks prior to expiration of the current contract.

Several diagnostic checks on the distributional properties of the data were performed, and the results are reported in Table 1. Based on unit root and cointegration tests, we find evidence that the spot and futures prices have common stochastic trends, implying that the error correction term in Equation (11) belongs in the model. This is consistent with both the theoretical results of Brenner and Kroner (1993) and with the empirical results of Baillie and Bollerslev (1989) and Hakkio and Rush (1989), who find evidence of cointegration between spot and forward prices for currencies. The cointegrating regressions used to conduct the cointegration tests invariably gave $\hat{\delta} \approx 1$, indicating that the basis, or the difference between the spot and futures prices, is stationary. Because this estimate is super consistent (Stock (1987)) and because covered interest rate parity says that δ should equal one (Brenner and Kroner (1993)), we impose $\delta = 1$ in Equation (11) throughout the ensuing analysis.⁴

³When a holiday occurs on a Thursday, Wednesday's observation is used in its place. Also, due to estimation problems with Thursday's data, the data for the Swiss franc are Friday's closing prices. All data were obtained from Dunn and Hargitt, Inc.

⁴This restriction is used by others in tests for market efficiency; see, for example, Baillie (1989) who imposes this restriction in his regressions and Wickens (1989) who argues that little is lost by imposing it.

TABLE 1
Data Description

Currency		$Z(t_{\alpha^*})$	$Z(t_{\alpha})$	Skewness	Kurtosis	B-J	$Q(24)$	$Q^2(24)$	ARCH(5)
BP	Spot	-2.90	-6.59	0.26	6.01	102.25	14.65	47.54	26.62
	Futures	-2.86		0.42	6.10	113.04	17.21	40.28	20.01
CD	Spot	-0.35	-6.07	-0.28	4.70	35.11	21.93	23.61	23.15
	Futures	-0.39		-0.14	4.04	12.71	24.02	18.59	10.16
DM	Spot	-2.43	-6.17	0.27	4.76	37.14	27.52	39.90	5.45
	Futures	-2.47		0.29	4.49	28.02	25.14	36.34	3.87
JY	Spot	-2.73	-6.08	0.72	7.28	223.46	14.97	23.11	6.33
	Futures	-2.75		0.77	7.16	215.63	14.35	20.41	5.04
SF	Spot	-2.67	-6.38	0.25	3.24	3.38	29.07	54.25	20.46
	Futures	-2.70		0.17	3.49	3.90	29.29	40.09	15.62
99% c.v.		-3.43	-4.07			9.21	42.98	42.98	15.09
95% c.v.		-2.86	-3.37			5.99	36.42	36.42	11.07

The spot data is the log of the weekly exchange rate, and the futures data is the log of the weekly futures rate. All unit root and cointegration tests are based on logarithms of the raw data, while the other statistics are based on differenced logs. $Z(t_{\alpha^*})$ is the Phillips and Perron (1988) test statistic for a unit root, with a truncation lag of 4. The null hypothesis is that a unit root exists. $Z(t_{\alpha})$ is the Engle and Granger (1987) test for cointegration. The null hypothesis is no cointegration, and the statistic is computed by applying the Phillips and Perron (1988) test to the residuals of the cointegrating regression

$$\ln S_t = \alpha + \delta \ln F_t + \epsilon_t$$

B-J is the Bera-Jarque test for normality. The statistic is

$$\text{B-J} = T \left[\frac{\text{skewness}^2}{6} + \frac{(\text{kurtosis} - 3)^2}{24} \right]$$

B-J is distributed χ^2_2 under the null of normality. $Q(24)$ is the Ljung-Box test for up to 24th order serial correlation in the raw data, and $Q^2(24)$ is the Ljung-Box test for up to 24th order serial correlation in the squared data. ARCH(5) is Engle's (1982) LM test for 5th order ARCH effects.

Further diagnostics on the logged-differenced data are also reported in Table 1. The unconditional distributions of ΔS_t and ΔF_t are nonnormal, as evidenced by high skewness, high kurtosis, and significant Bera-Jarque statistics. Tests for autocorrelation in the first moments using the $Q(24)$ statistic indicate that none is present in any of the exchange rates. Finally, tests for ARCH using both the $Q^2(24)$ and Engle's (1982) LM statistic generally support the hypothesis of time-varying variances. Overall, the existence of cointegration and the existence of ARCH suggest that the above procedure for finding the dynamic hedge has the potential to be an effective strategy.

IV. Comparison among Hedging Techniques

A. Model Estimation

In this section, we evaluate the potential usefulness of the proposed risk-minimizing hedging strategy by comparing it to three alternative strategies: the naive hedge, the conventional hedge, and a hedge from an error correction model (the *CI* hedge). To do this, the model in Equations (11) to (14) is estimated, first with restriction $H_{01} : a_s = b_s = a_f = b_f = \alpha_{1s} = \alpha_{1f} = 0$ imposed, which gives the conventional model, then with the restriction $H_{02} : a_s = b_s = a_f = b_f = 0$ imposed,

which gives the *CI* model, then with no restrictions imposed, which gives the conditional model. The results for the conventional and *CI* models are presented in Table 2. The *CI* model corrects for the misspecification in the conventional model arising from the cointegration between spot and futures prices, as evidenced by significant likelihood ratio test statistics. The hedge ratios, however, are similar to those computed from the conventional model.

Finally, the model is estimated with no restrictions, and the results are reported in Table 3. Likelihood ratio statistics (given in the bottom panel of Table 3) pitting this model against the conventional model and the *CI* model imply that in a statistical sense, the GARCH(1,1) model describes the depreciation rates of spot and futures better than the constant covariance models. They also suggest that the parameters a_s, \dots, b_f belong for each of the five currencies, which implies that the variances and covariances, and therefore the risk-minimizing hedge, are indeed changing through time. Diagnostics on the standardized residuals from the conditional models, presented in the bottom panel of Table 3, suggest that the models are well-specified. There is no evidence of remaining dynamics in the variance equations (the $Q^2(24)$ statistics are insignificant), and we fail to reject the null of constant conditional correlations (the CORR(6) and CORR(24) statistics are insignificant).

Notice also that the coefficients on the error correction terms in the *CI* model are consistent with those implied by the theoretical model in Brenner and Kroner (1993). In particular, contrary to the perceived wisdom in the current literature that if markets are efficient, then the ECT belongs in the spot equation with a coefficient of one,⁵ Brenner and Kroner (1993) show that if the spot rate follows a geometric Brownian motion, then covered interest rate parity implies that the coefficient on the ECT in the spot equation should be zero, while the coefficient in the futures equation should be one. For four of our five currencies (the DM is the exception), the coefficient on the ECT in the spot equation is insignificantly different from zero and significantly less than one at the 1-percent level. Also, for all five currencies, the coefficient on the ECT in the futures equation is not significantly different from one, and for three of the five it is significantly greater than zero.

Figure 1 plots the time-varying hedge ratios, which are calculated from Equation (15). The conditional hedge ratios are clearly changing as new information arrives at the market. To further examine the dynamics in the optimal hedge ratios, we performed simple time-series analysis on each of the hedge ratio series, and report the results in the bottom panel of Table 3. Tests for unit roots reported in the $Z(t_{\hat{\alpha}})$ row reveal that the hedge ratios are stationary. This is in sharp contrast to the results of Baillie and Myers (1991), who find that the hedge ratios for each of their six commodities are nonstationary. The implication is that, in contrast to commodity markets, currency market hedge ratios are mean reverting, implying that the impact of a shock to the hedge ratios eventually becomes negligible. Simple diagnostics on each of the five series (not reported) reveal that the hedge ratios follow AR(1) processes. The first-order serial correlation coefficients are presented in the ϕ row of Table 3. The DM hedge ratios are negatively serially

⁵See, for examples, Baillie and McMahon (1989) and the numerous examples cited therein.

TABLE 2
Maximum Likelihood Estimates for the Conventional and CI Models

	BP	CD	DM	JY	SF
α_{0s}	0.17 (1.68)	0.05 (1.54)	0.25 (2.89)	0.22 (2.49)	0.25 (2.30)
α_{0f}	0.16 (1.56)	0.05 (1.21)	0.25 (2.89)	0.21 (2.50)	0.25 (2.32)
α_{1s}	0.26 (1.27)	0.30 (2.74)	0.31 (1.13)	0.13 (0.46)	-0.07 (-0.27)
α_{1f}	0.67 (3.22)	0.47 (4.22)	0.75 (2.70)	0.49 (1.73)	0.39 (1.56)
c_{ss}	2.78 (19.64)	2.63 (20.23)	2.36 (18.71)	2.19 (23.67)	3.37 (13.63)
c_{sf}	2.77 (19.58)	2.61 (19.58)	2.32 (17.95)	2.12 (23.62)	3.33 (14.00)
c_{ff}	2.91 (19.75)	2.69 (18.64)	2.37 (16.99)	2.14 (22.39)	3.42 (14.47)
b^*	0.952 (-638.7)	0.875 (-196.8)	0.979 (-562.9)	0.991 (-542.4)	0.974 (-671.6)
ξ	29.42	62.56	51.26	61.38	64.08

The maximum likelihood estimates of the conventional and CI models are presented. The CI model is

$$s_t = \alpha_{0s} + \alpha_{1s}(\ln S_{t-1} - \ln F_{t-1}) + \epsilon_{st}$$

$$f_t = \alpha_{0f} + \alpha_{1f}(\ln S_{t-1} - \ln F_{t-1}) + \epsilon_{ft}$$

$$H_t = \begin{bmatrix} c_{ss} & c_{sf} \\ c_{sf} & c_{ff} \end{bmatrix}$$

The conventional model imposes the restriction $\alpha_{1s} = \alpha_{1f} = 0$. Asymptotic t -statistics are given in parentheses. b^* = c_{sf}/c_{ff} is the implied hedge ratio. L is the log-likelihood, and ξ is the likelihood ratio test for H_0 . $\alpha_{1s} = \alpha_{1f} = 0$. It is χ^2_2 and has 95-percent critical value 5.99.

TABLE 3
Maximum Likelihood Estimates for the Conditional Model

	BP	CD	DM	JY	SF
α_{0s}	0.01 (0.06)	-0.06 (-1.07)	0.62 (3.75)	0.27 (1.23)	0.23 (0.99)
α_{0f}	-0.18 (-1.18)	-0.17 (-2.79)	0.84 (4.96)	0.44 (2.03)	0.49 (2.14)
α_{1s}	0.32 (1.49)	0.39 (2.22)	0.91 (3.69)	0.14 (0.37)	0.04 (0.11)
α_{1f}	0.71 (3.14)	0.75 (4.01)	1.34 (5.39)	0.49 (1.35)	0.49 (1.40)
c_s	0.17 (1.75)	0.07 (2.53)	3.30 (6.26)	2.14 (1.91)	1.66 (1.67)
c_f	0.12 (1.91)	0.15 (2.28)	3.33 (5.12)	2.13 (1.26)	1.24 (1.59)
a_s	0.04 (2.90)	0.08 (3.10)	0.17 (3.57)	0.07 (1.98)	0.08 (1.84)
a_f	0.03 (2.45)	0.11 (3.20)	0.15 (2.89)	0.05 (1.46)	0.07 (1.75)
b_s	0.89 (19.05)	0.70 (7.43)	-0.33 (-2.81)	0.14 (0.34)	0.48 (1.75)
b_f	0.93 (31.52)	0.50 (2.76)	-0.33 (-1.86)	0.14 (0.21)	0.61 (2.73)
ρ	0.98 (509.0)	0.96 (251.8)	0.99 (720.8)	0.99 (919.9)	0.98 (623.1)
L	-589.39	-158.66	-517.10	-506.80	-632.51
ξ_{01}	98.58	76.30	91.66	71.20	78.18
ξ_{02}	36.02	46.88	30.28	19.94	14.10
CORR(6)	5.92	11.98	2.93	1.31	6.67
CORR(24)	12.28	21.56	30.38	18.43	31.20
$Q^2(24)$ (spot)	13.06	15.83	36.64	19.65	32.27
$Q^2(24)$ (futures)	13.20	21.18	34.88	17.33	28.85
$Z(t_{\hat{\alpha}})$	-3.21	-4.27	-23.01	-8.00	-5.71
ϕ	0.909	0.767	-0.188	0.417	0.642

This table presents the maximum likelihood estimates of the model

$$\begin{aligned}
 s_t &= \alpha_{0s} + \alpha_{1s}(S_{t-1} - F_{t-1}) + \epsilon_{st} \\
 f_t &= \alpha_{0f} + \alpha_{1f}(S_{t-1} - F_{t-1}) + \epsilon_{ft} \\
 H_t &= \begin{bmatrix} h_{s,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \\
 h_{s,t}^2 &= c_s + a_s \epsilon_{s,t-1}^2 + b_s h_{s,t-1}^2 \\
 h_{f,t}^2 &= c_f + a_f \epsilon_{f,t-1}^2 + b_f h_{f,t-1}^2
 \end{aligned}$$

Asymptotic t -statistics are given in parentheses. L is the log-likelihood, and ξ_1 is the likelihood ratio test statistic for H_{01} $\alpha_{1s} = \alpha_{1f} = a_s = a_f = b_s = b_f = 0$. It is χ_6^2 and has 95-percent critical value 12.59. ξ_2 is the likelihood ratio test statistic for H_{02} $a_s = a_f = b_s = b_f = 0$. It is χ_4^2 and has 95-percent critical value 9.49. CORR(6) and CORR(24) are the Box-Pierce test statistics for up to 6th and 24th order serial correlation in $\hat{v}_s \hat{v}_f$, where $\hat{v}_i = \hat{\epsilon}_{it} / \hat{h}_{it}$. They are distributed χ_6^2 and χ_{24}^2 under the null of constant correlations. $Q^2(24)$ is the Box-Pierce test for up to 24th order serial correlation in the squared standardized residuals. It is distributed χ_{24}^2 under the null of no serial correlation, and has 95-percent critical value 36.42. The first order serial correlation coefficients in the estimated hedge ratios are given by ϕ , and we reject the null hypothesis of a unit root in the estimated hedge ratios at the 95-percent level if $Z(t_{\hat{\alpha}}) < -2.86$.

correlated, meaning that in the absence of other shocks, a high hedge ratio this week will probably be followed by a low hedge ratio next week. This is difficult to justify economically, and may simply be a result of the sample period chosen. The other four hedge ratios are positively serially correlated. Hence, if one of these hedge ratios is large this week (relative to its long-run mean), then in the absence

of new shocks to the system, we would expect it to remain large next week, but it would be converging to its long-run mean.

B. Comparisons of Hedging Performance

In order to formally compare the performances of each kind of hedge, we construct the portfolios implied by the computed hedge ratios each week and calculate the variance of the returns to these portfolios over the sample, i.e., we evaluate

$$(16) \quad \text{Var}(s_t - b_t^* f_t),$$

where b_t^* are the computed hedge ratios. Table 4A reports the results. Within sample, our conditional hedge outperforms all other hedges considered for all currencies, with improvements averaging about 6 percent over the naive hedge and about 2½ percent over the conventional hedge. Not surprisingly, we also see from this table that the naive hedge, where spot market positions are matched dollar for dollar in the futures market, is the worst hedging strategy. One final observation is that the best hedging strategy of all the *constant* hedge models is the conventional model. This is expected because the conventional model explicitly solves for the hedge ratio, which minimizes the within-sample portfolio variance, and therefore its resulting within-sample variance must be smaller than any other constant hedge ratio strategy.

TABLE 4A
Comparisons of Within-Sample Hedging Effectiveness
Variance Comparisons

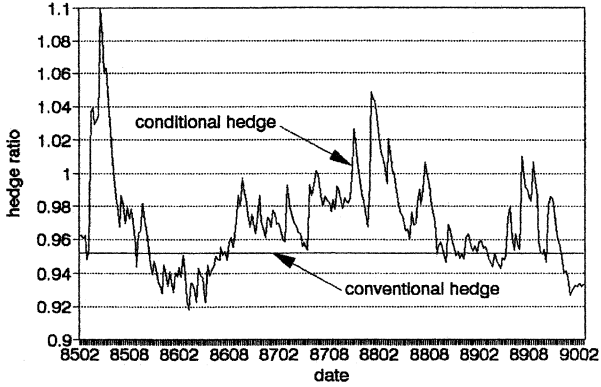
<i>Portfolio Variances</i>	BP	CD	DM	JY	SF
Naive Hedge ($b = 1$)	0 14003	0 04012	0 08666	0 07966	0 14364
Conventional Hedge ($b = b^*$)	0 13396	0 03508	0 08617	0 07963	0 14099
CI Hedge	0 13437	0 03508	0.08630	0 07965	0 14100
Conditional Hedge ($b = b_t^*$)	0 12976	0 03503	0 08221	0 07777	0 13786
<i>Percentage Variance Improvement of Conditional Hedge Compared to</i>	BP	CD	DM	JY	SF
Naive Hedge	7 33%	12 70%	5 14%	2 37%	4 02%
Conventional Hedge	3 13%	0.14%	4 60%	2 34%	2 22%
CI Hedge	3 43%	0 14%	4 74%	2 36%	2 23%

The sample variances of portfolio returns are computed from Equation (16) in the text, assuming that an investor used each of the hedging strategies discussed in the text. The percentage improvement of the conditional hedge over the other hedging strategies is also given.

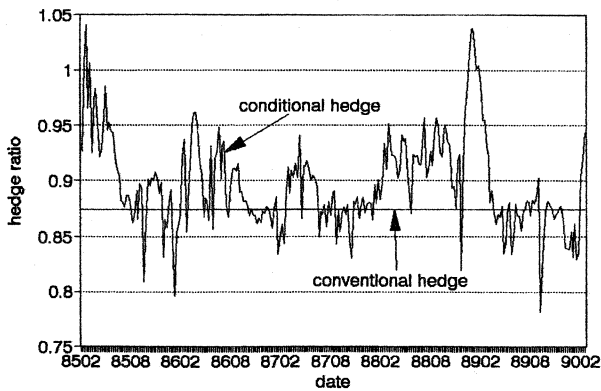
While these percentage improvements are not dramatic, they do not imply that the economic viability of our proposed strategy is questionable. To better understand the significance of these variance reductions, consider an investor with the mean-variance utility function given in Equation (3) and with degree of risk aversion $\gamma = 4$.⁶ If the expected returns to the hedged portfolio are zero and this

⁶This assumption on γ is in line with most empirical studies in the literature. For example, Chou (1988) estimates this parameter to be 4.5, Poterba and Summers (1986) estimate it to be 3.5, Grossman and Shiller (1981) estimate it to be 4, and Friend and Hasbrouck (1982) find it to be 6.

FIGURE 1
**British Pound
Hedge Ratios**



**Canadian Dollar
Hedge Ratios**



**German Mark
Hedge Ratios**

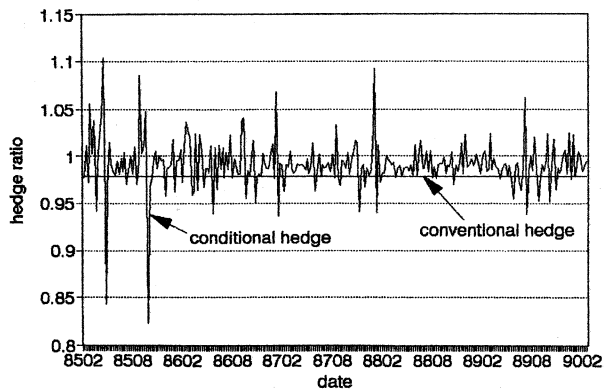
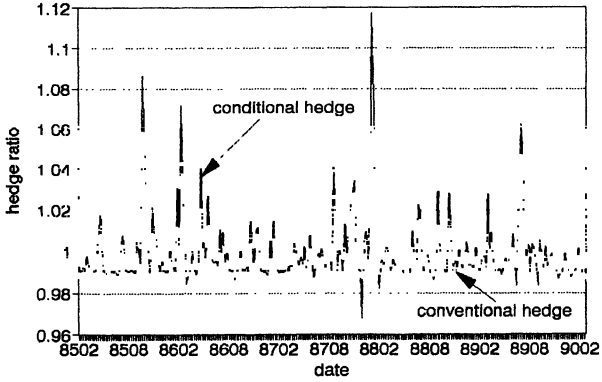


FIGURE 1 (continued)

Japanese Yen Hedge Ratios



Swiss Franc Hedge Ratios

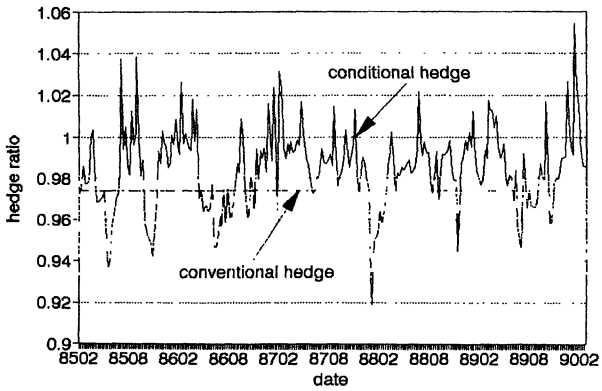


TABLE 4B
 Comparisons of Within-Sample Hedging Effectiveness
 Utility Comparisons

Hedge	Trans Cost	BP	CD	DM	JY	SF
Conventional		-109.29 (0)	-30.52 (0)	-73.29 (0)	-67.29 (0)	-116.89 (0)
Conditional	$y = 0.0100$	-105.17 (18)	-30.13 (20)	-71.28 (82)	-66.54 (30)	-115.82 (46)
Conditional	$y = 0.0125$	-105.40 (15)	-30.45 (20)	-71.54 (72)	-67.00 (12)	-115.99 (35)
Conditional	$y = 0.0150$	-105.58 (15)	-30.60 (14)	-71.76 (63)	-67.03 (12)	-116.28 (33)

This table presents the utility of the portfolio constructed by assuming that an investor rebalances his portfolio only when the potential gains in utility from the reduced variance offset the transactions costs that must be incurred. The utility of this portfolio is computed from Equation (8) in the text. y is the transactions cost in percent. The number of portfolio rebalancings is given in parentheses.

hedger invested in the conventional hedge portfolio for the British pound, he would have had an “average utility” of $U(x) = -4(0.13396) = -0.53584$ each week. On the other hand, if he had invested in the conditional hedge, his “average utility” would have been $U(x) = -y - 4(0.12976) = -y - 0.51904$, where y represents the reduced returns caused by the transactions costs incurred. Therefore, the investor’s utility increases by $(0.01680 - y)$ if he uses the conditional hedge, so the conditional strategy will be preferred if $y < 0.01680$. But for a highly leveraged investment strategy such as foreign currency futures hedging, the transaction costs are low. One round trip (one buy and one sell) costs only \$10–\$15, implying $y \approx \frac{10}{100000} = 0.01$ percent. Therefore, even after accounting for transactions costs, and even though the percentage variance reduction is not large, the conditional hedge would result in an improvement in utility for an investor with a mean-variance utility function and $\gamma = 4$. Similar conclusions are found for the DM, the SF, and for the JY if $\gamma = 5\frac{1}{2}$.

But perhaps an even better way to understand the economic usefulness of our proposed model is to consider the following alternative hedging strategy, which is also based on our conditional model: Investors will only rebalance their portfolios when the benefits to doing so offset the costs of doing so, or in other words, when the increased expected utility from rebalancing is great enough to offset the transactions costs that will have to be incurred. Once again, assume that y is the percentage returns that an investor “loses” due to transactions costs every time he goes to the futures market. The returns to his portfolio are then $x_{t+1} = s_{t+1} - b_t^* f_{t+1} - y$ if he rebalances, and $x_{t+1} = s_{t+1} - b_t^* f_{t+1}$ if he does not rebalance, where b_t^* is the hedge ratio from the most recent rebalancing. Therefore, the expected return to his portfolio is $-y$ if he rebalances and 0 if he does not. The conditional variance of his portfolio if he rebalances is $\sigma_t^2(x_{t+1}) = h_{ss,t+1} - 2b_t^* h_{sf,t+1} + b_t^{*2} h_{ff,t+1}$, and the conditional variance if he does not rebalance is $\sigma_t^2(x_{t+1}) = h_{ss,t+1} - 2b_t^* h_{sf,t+1} + b_t^{*2} h_{ff,t+1}$. Therefore, a mean-variance expected utility-maximizing investor will rebalance at time t if and only if

$$-y - \gamma(h_{ss,t+1} - 2b_t^* h_{sf,t+1} + b_t^{*2} h_{ff,t+1}) > h_{ss,t+1} - 2b_t^* h_{sf,t+1} + b_t^{*2} h_{ff,t+1}.$$

If an investor in British pounds followed this strategy during our sample period and used the conditional model to obtain estimates of b_t^* , $h_{ss,t+1}$, $h_{sf,t+1}$, and $h_{ff,t+1}$, he would have rebalanced only 18 times during the 263-week period covered in our sample (assuming $y = 0.01$), and his total expected utility over the 263 weeks

would have been -105.17 . On the other hand, if he had avoided all the transactions costs and invested in the conventional hedge, his utility function would have been -109.29 . So, clearly, a mean-variance expected utility-maximizing investor would prefer the dynamic strategy to the conventional strategy even though the percentage variance reduction is not large. Table 4B contains the results of this analysis for all the currencies for different values of γ . Notice that even for the Canadian dollar, where the percentage variance reduction is almost zero, a mean-variance utility-maximizing investor would still generally prefer our dynamic strategy, even after taking transactions costs into account.⁷

Of course, investors are less concerned with how well they could have done in the past if they had used a different strategy than with how well they can do in the future if they use a different strategy. So out-of-sample performance is a better way to evaluate our proposed hedging strategy. To compare the out-of-sample hedging performances, we withhold one-half of the sample (132 observations, or after July 17, 1987) and estimate the conventional, *CI*, and conditional models using only the data up to July 10, 1987. Using each of these three estimated models, we *forecast* the hedge for the following week (July 10 to July 17) by computing the one-period forecast of the covariance divided by the one-period forecast of the variance. The following week (July 17) the exercise is repeated, with the new observation included in the data set, i.e., a hedge is forecasted for the week of July 17 to July 24, using data available on July 17. We continue updating the models and forecasting the hedge ratios on a weekly basis until the end of our data set, giving us a series of 133 forecasted portfolios. Table 5A gives the out-of-sample variance results. The conditional hedge again outperforms all the other hedges for all currencies, except this time it is not able to show improvement over the conventional hedge for the British pound. On average, however, the conditional hedge results in variance reductions of about $4\frac{1}{2}$ percent compared to the naive hedge and about $1\frac{1}{2}$ percent compared to the conventional hedge. Again, to get a better measure of the economic usefulness of our model, we make the same utility comparisons as above, assuming an investor rebalances only if it pays to do so. The results are reported in Table 5B, and again our strategy results in higher utility than the other strategies, meaning that even after accounting for transactions costs, investors would still prefer our dynamic strategy.

V. Conclusions

Evidence presented in this paper and elsewhere that return distributions of many assets are time varying and that asset prices are cointegrated raises concerns regarding the risk reduction properties of conventional hedging models, which ignore both of these problems. This study presents an alternative hedging model for calculating risk-minimizing hedge ratios in foreign currency futures, and compares the hedging effectiveness of this conditional hedging model with that of a conventional method. Both within-sample and out-of-sample evidence presented in this paper indicates that the hedging strategy proposed here is potentially superior to

⁷The qualitative nature of these results is unchanged by reducing γ to two. Clearly, however, these results are contingent on a mean-variance utility function. A different utility function would result in different hedge ratios, and perhaps different rankings of the hedging strategies.

TABLE 5A
 Comparisons of Out-of-Sample Hedging Effectiveness
 (July 10, 1987, to February 23, 1990)

Variance Comparisons					
<i>Portfolio Variances</i>	BP	CD	DM	JY	SF
Naive Hedge ($b = 1$)	0.10858	0.03567	0.07526	0.11101	0.15854
Conventional Hedge ($b = b^*$)	0.10241	0.03316	0.07483	0.11154	0.15479
CI Hedge	0.10364	0.03278	0.07547	0.11228	0.15257
Conditional Hedge ($b = b^*$)	0.10339	0.03162	0.07391	0.11048	0.15313
<i>Percentage Variance Improvement of Conditional Hedge Compared to:</i>					
	BP	CD	DM	JY	SF
Naive Hedge	4.79%	11.34%	1.79%	0.48%	3.41%
Conventional Hedge	-0.95%	4.64%	1.24%	0.96%	1.07%
CI Hedge	0.24%	3.54%	2.07%	1.60%	-0.37%

The sample variances of portfolio returns are computed from Equation (16) in the text, assuming that an investor forecasted the next week's hedge ratio using only data available to him at that time. The forecasts are made with each of the different hedging models discussed in the text. The percentage improvement of the conditional hedge over the other hedging strategies is also given.

TABLE 5B
 Comparisons of Out-of-Sample Hedging Effectiveness
 (July 10, 1987, to February 23, 1990)

Utility Comparisons						
Hedge	Trans Cost	BP	CD	DM	JY	SF
Conventional	$y = 0.0100$	-48.83 (2)	-15.01 (1)	-36.02 (2)	-30.03 (1)	-55.05 (2)
Conventional	$y = 0.0125$	-48.91 (1)	-15.01 (1)	-36.04 (1)	-30.03 (1)	-54.97 (1)
Conventional	$y = 0.0150$	-48.91 (1)	-15.02 (1)	-36.04 (1)	-30.03 (1)	-54.97 (1)
Conditional	$y = 0.0100$	-47.16 (7)	-14.66 (10)	-35.79 (36)	-29.64 (10)	-54.70 (34)
Conditional	$y = 0.0125$	-47.18 (7)	-14.74 (6)	-35.96 (34)	-29.66 (10)	-54.82 (33)
Conditional	$y = 0.0150$	-47.23 (6)	-14.78 (6)	-36.04 (29)	-29.75 (10)	-54.95 (27)

This table presents the utility of the portfolio constructed by assuming that an investor rebalances his portfolio only when the forecasted potential gains in utility (using only data available to the investor at the time the forecast is made) from the reduced variance offset the transactions costs that must be incurred. The utility of this portfolio is computed from Equation (8) in the text. y is the transactions cost in percent. The number of portfolio rebalancings is given in parentheses.

the other methods, in that it offers investors an improved ability to manage foreign exchange exposure. We find that our strategy results in reduced portfolio variance both within-sample and out-of-sample. We also use this model as the foundation for a dynamic hedging strategy in which an investor rebalances only if the potential utility gains from rebalancing offset the losses due to transactions costs, and find that this strategy is also superior to the conventional strategy. Therefore, our model gives an improved hedging strategy even after accounting for transactions costs.

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