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MULTIVARIATE GARCH MODELING OF ASSET RETURNS[1]

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Abstract: This paper studies asymmetric responses of covariances to return shocks. Asymmetric/leverage effects have been found in variances but few studies have examined such effects in covariances, even though they might have important implications for portfolio management and hedging. We propose a robust conditional moment test to detect the presence of such asymmetric effects in the covariances. We also introduce a general dynamic covariance matrix model, which nests many of the existing multivariate GARCH models and addresses covariance asymmetries. To illustrate the tests and the new model, we apply them to weekly returns from a large firm and a small firm portfolio. Our results suggest that bad news about large firms can cause volatility in both small firm returns and large firm returns. The conditional covariance between large firm returns and small firm returns will also increase following bad news about large firms. In contrast, news about small firms has a minimal effect on variances and covariances.

I. INTRODUCTION

Recent studies on the time series properties of stock return volatility find that volatility is predictable and is affected by the magnitude and direction of past return shocks. See Bollerslev, Chou and Kroner (1992) for a survey. Specifically, large return shocks lead to high subsequent volatility, with negative return shocks inducing higher subsequent volatility than positive return shocks of the same magnitude (Black, 1976; French, Schwert and Stambaugh, 1987; Nelson, 1991; Pagan and Schwert, 1990; Engle and Ng, 1993). To capture such empirical regularities, many volatility models have been developed in the literature and used in the valuation of assets. For instance, Kuwahara and Marsh (1992) use the GARCH and EGARCH models to price Japanese Equity Warrants, and Amin and Ng (1994) compare the performance of several GARCH models in pricing individual stock options. These papers generally find that the asymmetric/leverage effects in volatility is important in option valuation.

While relatively few studies have examined the existence of asymmetric effects in the covariances, it is conceivable that such effects might exist, possibly for similar reasons as for volatility asymmetries. For instance, if the asymmetry in volatility is caused by a leverage effect – an increase in the riskiness of the stock due to an increase in the debt/equity ratio of the firm following a price drop – then the change in financial leverage in the firm might also change the degree of comovement between its stock return and other stock returns. As another possibility, if the asymmetric effect in volatility is caused by an increase in the amount of information flow following bad news, then the covariance between stock returns will also be affected because there would be a different relative rate of information flow across firms. Furthermore, since covariances play an important role in portfolio selection, risk management and the pricing of derivative assets, these effects could have important investment implications.

This paper has three contributions. First, given the lack of diagnostic tests for multivariate models with time varying variances and covariances, we introduce a robust conditional moment test to detect the presence of such asymmetric effects in covariances. Second, we introduce a general model which nests the four most popular time varying covariance models: the VECH model of Bollerslev, Engle, and Wooldridge (1988), the constant correlation model of Bollerslev (1990), the factor ARCH model of Engle, Ng, and Rothschild (1990), and the BEKK model of Engle and Kroner (1995). Third, we work out a natural generalization of this encompassing model to allow for asymmetric/leverage effects in variances and covariances. This asymmetric dynamic covariance matrix model nests various asymmetric extensions of the four existing models.

To illustrate the models and the new tests, we apply them to the bivariate system of weekly large firm and small firm portfolio returns used in Conrad, Gultekin and Kaul (1991). The sample period is from July 1962 to December 1988, for a total of 1371 weekly observations. We find that all four existing models are misspecified, especially in the dynamics of the

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covariances. Using our proposed model, we find that bad news about large firms causes increased volatility in both small firm and large firm returns. The conditional covariance between small and large firm returns also increases after bad news about large firms. In contrast, news about small firms has a minimal effect on variances and covariances.

2. TIME VARYING COVARIANCE MODELS

In an extension of the Capital Asset Pricing Model to allow for time varying betas, Bollerslev, Engle and Wooldridge (1988) model the time varying variances and covariances of asset returns with a Box-Jenkins ARMA type specification for the squares and cross products of unexpected asset returns. This specification is named the VECH model in the literature. It has also been applied by Giovannini and Jorion (1989) to study foreign exchange volatility, by Bodurtha and Mark (1991) to reevaluate CAPM using U.S. equity market data, and by Baillie and Myers (1991) to compute optimal hedge ratios in commodity spot and futures markets.

Let R_{it}, $i=1,\ldots,N$, be the rates of return of asset i at time t. Let Ψ_{t-1} be an information set containing the history of all variables up to time $t-1$. Since investors know Ψ_{t-1} when they make their investment decision at time $t-1$, the relevant measures of expected return, return variability and comovement are the mean returns, the variances, and the covariance or correlation conditional on Ψ_{t-1}. Let $\mu_{it} \equiv E_{t-1}R_{it}$, $h_{iit} \equiv \text{var}_{t-1}(R_{it})$, and $h_{ijt} \equiv \text{cov}_{t-1}(R_{it}, R_{jt})$, where $E_{t-1}(\bullet)$ is the conditional expectation operator, $\text{var}_{t-1}(\bullet)$ is the conditional variance operator, and $\text{cov}_{t-1}(\bullet)$ is the conditional covariance operator. Further, let $H_t \equiv [h_{ij,t}]$ be the conditional covariance matrix of asset returns at time t. Under the VECH model, H_t is modeled as follows:

$$(1) \quad h_{ij,t} = \omega_{ij} + \beta_{ij} h_{ij,t-1} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1}$$

for all $i, j = 1,\cdots,N$, where $\varepsilon_{i,t-1} \equiv R_{i,t-1} - \mu_{i,t-1}$ is the stock return shock to asset i at time t-1.

To ensure the positive definiteness of the estimated variance covariance matrix of asset returns, restrictions have to be imposed on the parameters to ensure that $h_{ii,t}(\bullet)$ is a positive function and the conditional correlation $\rho_{ij,t} = h_{ij,t}\big/\sqrt{h_{ii,t}h_{jj,t}}$ is a bounded function in the interval (1,–1). A sufficient condition is that $\omega_{ii} > 0$, $\beta_{ii} \geq 0$, $\alpha_{ii} \geq 0$, $\omega_{ij} \leq \sqrt{\omega_{ii}\omega_{jj}}$, $\beta_{ij} \leq \sqrt{\beta_{ii}\beta_{jj}}$, and $\alpha_{ij} \leq \sqrt{\alpha_{ii}\alpha_{jj}}$. Because the number of restrictions increases exponentially with the number of assets, it is hard to keep track of all the parameter values, even for relatively small systems, that guarantee positive definiteness of the estimated variance covariance matrix.

To overcome this problem, Engle and Kroner (1995) propose using quadratic forms to model the variance and covariance functions. Their model, called the BEKK model, has been applied to model the time varying variances and covariances of different size based stock portfolios by Conrad, Gultekin and Kaul (1991), of international stock returns by Chan, Karolyi and Stulz (1992), and of spot and futures returns by Baillie and Myers (1991).

Let $\varepsilon_{t-1} = (\varepsilon_{1t-1},\ldots,\varepsilon_{Nt-1})'$ be the vector of return shocks at time $t-1$, and let C, A, and B be $N\times N$ matrices. The BEKK model takes the form

$$(2) \quad H_t = C'C + B'H_{t-1}B + A'\varepsilon_{t-1}\varepsilon'_{t-1}A.$$

This model allows the conditional covariance matrix of asset returns to be determined by the outer product matrices of the vector of past return shocks. Given that each term on the right hand side of (3) is expressed as a quadratic form, the positive definiteness of the conditional covariance matrix of asset returns is guaranteed provided that the null spaces of C and B intersect only at the origin (Engle and Kroner, 1995). A sufficient condition for this to hold is that either C or B be full rank.

While the positive definiteness of the estimated conditional covariance matrix is ensured, the number of parameters in the BEKK model is large especially for a system vith many asset return series. Alternatives which guarantee positive definiteness and fewer parameters have therefore been proposed. One is the Factor ARCH model (FAC) of Engle (1987), Engle, Ng and Rothschild (1990) and Ng, Engle and Rothschild (1992). Let Ω be an $N\times N$ positive definite symmetric matrix, λ and w be $N\times 1$ vectors, and α and φ be scalars. The specification for a one factor ARCH model is

$$(3) \quad H_t = \Omega + \lambda\lambda'\left[\varphi\cdot w'H_{t-1}w + \phi\cdot(w'\varepsilon_{t-1})^2\right].$$

This can be viewed as a special case of the BEKK model, in which the A and B matrices are rank one and share an eigenvector. Essentially, the Factor ARCH model assumes that the conditional variances and covariances of the asset returns are functions of the conditional variances of a portfolio return which follows a GARCH type process. Let $R_{pt} \equiv w'R_t$, where $R_t = (R_{1t}, \ldots, R_{Nt})'$, be the returns to a portfolio formed with a weight vector w. The time $t-1$ return shock of this portfolio is $\varepsilon_{p,t-1} = w'\varepsilon_{t-1}$ and the time t conditional variance of this portfolio is $h_{pt} = w'H_t w$. The one factor ARCH model can be rewritten in the following alternative form:

(4a) $h_{ijt} = \sigma_{ij} + \lambda_i \lambda_j \cdot h_{pt} \qquad \text{for all } i, j = 1, \ldots, N$

(4b) $h_{pt} = \omega_p + \varphi \cdot h_{p,t-1} + \phi \varepsilon^2_{p,t-1}$

where Ω_{ij} is the $(i,j)^{\text{th}}$ element of Ω, $\omega_p \equiv w'\Omega w$, and $\sigma_{ij} \equiv \Omega_{ij} - \lambda_i \lambda_j \omega_p$.

In Schwert and Seguin (1990) and the one factor case in Ng, Engle and Rothschild (1992), R_{pt} is taken to be the market return. In other words, the entire conditional covariance matrix of asset returns is driven by the conditional variance of the market portfolio. Like the BEKK model, the estimated conditional covariance matrix of asset returns is positive definite as long as the constant part of the conditional covariance matrix is positive definite.

A second way to parsimoniously model the time series behavior of the conditional covariance matrix of asset return is the constant correlation model (CCOR) suggested by Bollerslev (1990). This model restricts the conditional covariance between two asset returns to be proportional to the product of the conditional standard deviations of the asset returns. In this way the conditional correlation coefficient of the two asset returns is time invariant. Specifically, the model is:

(5a) $h_{iit} = \omega_{ii} + \beta_{ii} h_{ii,t-1} + \alpha_{ii} \varepsilon^2_{i,t-1} \qquad \forall i = 1, \ldots, N$

(5b) $h_{ijt} = \rho_{ij} \cdot \sqrt{h_{iit} h_{jjt}} \qquad \forall i \neq j.$

Bollerslev (1990) applies this model to study time varying volatility in exchange rates. Ng (1991) applies this model to study a conditional CAPM with time varying variances and covariance, and Kroner and Claessens (1991) and Kroner and Sultan (1993) apply this model to obtain better hedge ratios in currency markets. Chan, Chan, and Karolyi (1991) use a modified version of this model to study the volatility relationship between stock index spot and futures markets.

To systematically analyze the asymmetric properties of time varying covariance matrix models, we define the following effects, where ${}_k\varepsilon_{t-1}$ is the $(N-1)\times 1$ vector of return shocks at time $t-1$, excluding $\varepsilon_{k,t-1}$:

[1] Own volatility asymmetry

A covariance matrix function $H_t = Q(\varepsilon_{t-1}, H_{t-1}) = [q_{ij}(\varepsilon_{t-1}, H_{t-1})]$ exhibits own volatility asymmetry if for some i, $q_{ii}({}_i\varepsilon_{t-1}, \varepsilon_{i,t-1}, H_{t-1}) \neq q_{ii}({}_i\varepsilon_{t-1}, -\varepsilon_{i,t-1}, H_{t-1})$.

[2] Cross volatility asymmetry

A covariance matrix function $H_t = Q(\varepsilon_{t-1}, H_{t-1}) = [q_{ij}(\varepsilon_{t-1}, H_{t-1})]$ exhibits cross volatility asymmetry if for some i and for some $j \neq i$, $q_{ii}({}_j\varepsilon_{t-1}, \varepsilon_{j,t-1}, H_{t-1}) \neq q_{ii}({}_j\varepsilon_{t-1}, -\varepsilon_{j,t-1}, H_{t-1})$.

[3] Covariance asymmetry

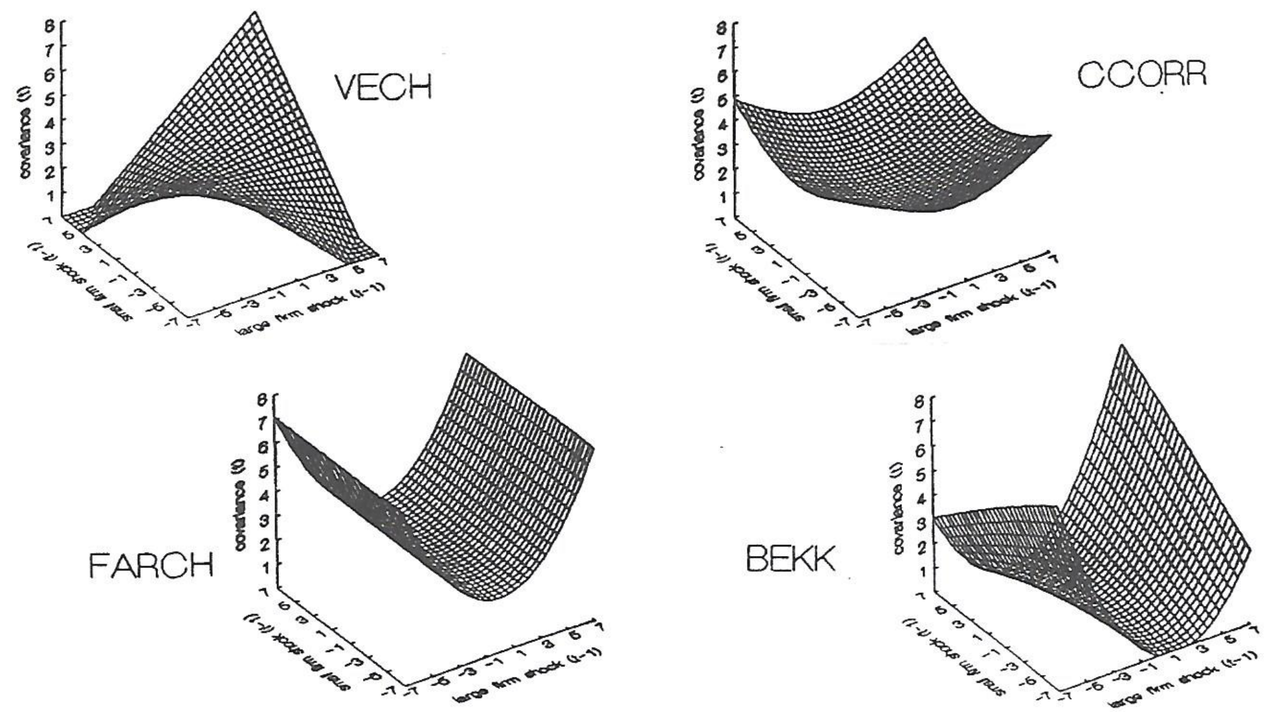
A covariance matrix function $H_t = Q(\varepsilon_{t-1}, H_{t-1}) = [q_{ij}(\varepsilon_{t-1}, H_{t-1})]$ exhibits covariance asymmetry if for some i and for some $j \neq i$, $q_{ij}({}_j\varepsilon_{t-1}, \varepsilon_{j,t-1}, H_{t-1}) \neq q_{ij}({}_j\varepsilon_{t-1}, -\varepsilon_{j,t-1}, H_{t-1})$.

In simple terms, own variance asymmetry means that the conditional variance of an asset is affected by the sign of the asset's own return shock. Cross variance asymmetry means that the conditional variance of an asset is affected by the sign of the return shock of another asset. Cross variance asymmetry is in fact an extension of the idea of volatility spillovers in Hamao, Masulis, and Ng (1990) and Chan, Chan, and Karolyi (1992), with good or bad news about one asset having different effects on the volatility of another asset. Finally, covariance asymmetry means that the covariance between two assets is affected by the sign of the return shock of at least one of the two assets. The properties of the four time varying covariance matrix models with respect to these asymmetric effects are summarized in the following table:

Table 1: Model Summaries

Model:	own-variance asymmetry	cross-variance asymmetry	covariance asymmetry
VECH	no	no	yes
BEKK	yes	yes	yes
FAC	yes	yes	yes
CCOR	no	no	no

Figure 1: Covariance News Impulse Response Surfaces from the Four Basic Models



To further describe these differences between the models, we define "news impact surfaces," which are three dimensional graphs of the current conditional covariance (variance) plotted against trailing large and small firm shocks, holding the past conditional variances and covariances constant at their in-sample averages. The graph is a bivariate generalization of the news impact curve in Engle and Ng (1993). More precisely, let q_{t-1} be the vector of inputs (known at time $t-1$) into h_{ijt}, excluding $\varepsilon_{i,t-1}$ and $\varepsilon_{j,t-1}$, and let Q be the unconditional mean of q_{t-1}. The news impact surface for h_{ijt} is the three dimensional graph of

$$h_{ijt} = h_{ij}\left(\varepsilon_{i,t-1}, \varepsilon_{j,t-1} \middle| q_{t-1} = Q\right).$$

Variance news impact surfaces reveal own variance asymmetry and cross variance asymmetry, while covariance news impact surfaces reveal covariance asymmetry.

Figure 1 shows the effects of return shocks of different sign and magnitude on the covariance between large and small firm portfolios implied by these four models.[2] The figures clearly illustrate that the different models imply different news impact surfaces for the covariances, even though the models are fitted to the same dataset. For example, the CCOR model implies that bad news to the small firm coupled with good news to the large firm causes increased covariances, while the VECH model implies that this causes decreased covariances. These kinds of differences have important implications on the computation of the optimal portfolio weights, optimal hedge ratios, and the betas of the securities in asset pricing problems. Therefore, we must question which, if any, of these models correctly specify the covariances, or equivalently, which, if any, of the covariance news impact surfaces are good descriptions of the data.

[2] Since our main focus is on the effect of last period's return shocks on current volatility, we simply model the mean of the return vector as a 10th order vector autoregression (VAR) with 10 lags of a threshold term. The threshold term is added to make sure that any asymmetry found in the variances and covariances is not caused by a misspecification in the mean. We do the estimation in two steps: first we estimate the mean equation to get the residuals, then we estimate the conditional covariance matrix parameters jointly using maximum likelihood, treating the residuals as observable data. The block diagonality of the information matrix guarantees that consistency is not lost in such a procedure. The estimation results (not reported) are available from the authors.

3. ROBUST CONDITIONAL MOMENT TESTS[3]

To test the validity of a model, we propose to use robust conditional moment tests (Wooldridge, 1990) to study how well the implied news response surfaces match the data. Conditional moment tests are distributed χ_k^2 under the null hypothesis that the "generalized residual" has mean zero, conditional on k "misspecification indicators". The generalized residual can be any function with mean zero if the model is correct, and the misspecification indicator is the direction(s) in which the test has maximum power. For our purposes, the generalized residual is the distance between the implied news impact surface and the realized data. Specifically, it is $u_{ij,t} = \varepsilon_{it}\varepsilon_{jt} - h_{ijt}$ if we are examining the covariance news impact surface, and $u_{ii,t} = \varepsilon_{it}^2 - h_{ijt}$ if we are examining the variance news impact surface. It is impractical to define misspecification indicators that test if each point on the surface is correct, so we propose to use indicators that examine only a limited set of points on the news impact surface. For most empirical applications, if the news impact surface is the wrong height somewhere, it is likely to be reflected in one of the following directions (cf. Engle and Ng, 1993):

Sign misspecification indicators:

The model might systematically over or underpredict covariances (variances) after good news or bad news. If so, the misspecification indicators

$$m_1^i = I(\varepsilon_{i,t-1} < 0),$$

where $I(\bullet)$ is the indicator function which takes the value one if the argument in parentheses is true, should be significant.

Quadrant misspecification indicators:

The model might systematically over or underpredict covariances (variances) in any of the four quadrants $(\varepsilon_1 > 0, \varepsilon_2 > 0)$, . . . , $(\varepsilon_1 < 0, \varepsilon_2 < 0)$. If so, at least one of the misspecification indicators

$$m_2^{-,-} = I(\varepsilon_{1,t-1} < 0, \varepsilon_{2,t-1} < 0)$$
$$m_2^{+,-} = I(\varepsilon_{1,t-1} > 0, \varepsilon_{2,t-1} < 0)$$
$$m_2^{-,+} = I(\varepsilon_{1,t-1} < 0, \varepsilon_{2,t-1} > 0)$$
$$m_2^{+,+} = I(\varepsilon_{1,t-1} > 0, \varepsilon_{2,t-1} > 0)$$

should be significant.

Size-sign misspecification indicators:

The model might systematically over or underpredict covariances (variances) following unexpectedly large or small return shocks of different signs. A set of misspecification indicators which tests for this is

$$m_3^{ii} = \varepsilon_{i,t-1}^2 I(\varepsilon_{i,t-1} < 0)$$
$$m_3^{ij} = \varepsilon_{i,t-1}^2 I(\varepsilon_{j,t-1} < 0)$$
$$m_3^{ji} = \varepsilon_{j,t-1}^2 I(\varepsilon_{i,t-1} < 0)$$
$$m_3^{jj} = \varepsilon_{j,t-1}^2 I(\varepsilon_{j,t-1} < 0).$$

Table 2 below reports the robust conditional moment test results for misspecification in the covariances from the four standard covariance models discussed above. The first variable in the model is the small firm portfolio returns, and the second variable is the large firm portfolio returns. Blank entries in the table mean that the test was insignificant from zero at the 5% level.

Table 2: Robust Conditional Moment Tests

	VECH	CCOR	BEKK	FAC
m_1^1	4.85	5.17	4.85	4.91
m_1^2	16.22	17.63	16.27	16.34
$m_2^{-,-}$	5.88	10.95	6.45	6.40
$m_2^{-,+}$		6.84		
$m_2^{+,-}$			4.10	
$m_2^{+,+}$	11.09	10.46	12.67	12.06
m_3^{11}				
m_3^{12}	3.99	6.45	5.54	5.01
m_3^{21}		4.03	5.40	4.24
m_3^{22}	4.41	4.95	6.29	5.11

The covariance equations from all four basic models are misspecified because they are unable to capture asymmetric effects in the covariances. Both sign misspecification indicators (m_1^1 and m_1^2) are different from zero for all the models, suggesting that bad news (negative residuals) have different effects on covariances than good news (positive residuals). This is corroborated by the sign-size misspecification indicators (m_3^{ij}), though the insignificant m_3^{11} implies that the model does not misspecify how small firm returns affect covariances when the small firm news is bad. Finally, the significance of $m_2^{-,-}$ and $m_2^{+,-}$ means

[3] See Kroner and Ng (1995) for a detailed discussion of robust conditional moment tests in the Multivariate GARCH context

that the models misspecify how covariances respond to bad news about the large firm portfolio, independent of the sign of the news to the small firm portfolio.

4. NEW MODELS

These test results strongly suggest the existence of asymmetric effects in the covariances and variances that none of the existing models can account for. A more general model is needed that can explicitly capture the asymmetries. Instead of working on extensions for each of the four models and then comparing the large number of possible extensions, we follow a more structured approach. First, we will introduce a general dynamic covariance matrix model that can nest all four models. Then, we will generalize this model to include asymmetric effects. The resulting asymmetric covariance matrix model encompasses various asymmetric extensions of the four models. The specification of the basic encompassing model is as follows:

General Dynamic Covariance (GDC) Matrix Model

Let a_i and b_i be $N\times 1$ vectors. The GDC model is

$$h_{ij,t} = \begin{cases} \theta_{ij,t} & \text{if } i=j \\ \phi_{ij}\theta_{ijt} + \rho_{ij}\sqrt{\theta_{iit}\theta_{jjt}} & \text{if } i\neq j \end{cases}$$

where

$$\theta_{ijt} \equiv \omega_{ij} + b_i' H_{t-1} b_j + a_i'\varepsilon_{i,t-1}\varepsilon_{j,t-1}' a_j.$$

In matrix notation, this is

$$H_t = \Omega + \Phi \circ A'\varepsilon_{t-1}\varepsilon_{t-1}' A + B' H_{t-1} B + \Lambda_{t-1} C \Lambda_{t-1},$$

where A and B are $N\times N$ matrices with columns a_i and b_i, $i=1,\ldots N$, respectively, Φ is a symmetric $N\times N$ matrix with ones along the diagonal and ϕ_{ij} off the diagonal, C is a diagonal matrix with zeros on the diagonal and ρ_{ij} off the diagonal, and $\Lambda_{t-1} \equiv \text{diag}\left\{\sqrt{h_{11,t}},\cdots,\sqrt{h_{NN,t}}\right\}$.

Parameter restrictions necessary to ensure that variances are positive and correlations are between −1 and 1 are that $\rho_{ij}\in(-1,1)$ for all $i\neq j$ and that $|\phi_{ij}|\leq 1-|\rho_{ij}|$. See Kroner and Ng (1995) for proof.

It is straightforward to show that this model nests the BEKK, CCOR and FAC covariance models. For example, if $\phi_{ij}=0$ for all i and j, the model reduces to a CCOR model in which the coefficients are restricted to be positive. Similarly, if $\rho_{ij}=0$ and $\phi_{ij}=1$ for all i and j, the model reduces to the BEKK model. And as discussed in section 2 above, further restrictions on the BEKK model will yield the FAC model. Furthermore, the GDC model nests a positive definite form of the VECH model. Specifically, if $\rho_{ij}=0$ for all $i\neq j$ and if $a_i=\alpha_i\iota_i$ and $b_i=\beta_i\iota_i$ where ι_i is the i^{th} column of an $N\times N$ identity matrix and α_i and β_i are scalars, then the GDC model is a special case of the diagonal VECH model in which the parameters are restricted to ensure positive definiteness. See Kroner and Ng (1995) for further discussion and proof of these special cases.

To generalize the above model to allow for asymmetric effects in the variances and covariances, we follow the direction of Glosten, Jagannathan, and Runkle (1993) and Zakoian (1994). In particular, we add new terms related to $\min(\varepsilon_{i,t-1},0)$. The model is defined as follows:

Asymmetric Dynamic Covariance (ADC) Matrix Model

The ADC model is similar to the GDC model, except that the definition of θ_{ijt} is replaced with

$$\theta_{ijt} \equiv \omega_{ij} + b_i' H_{t-1} b_j + a_i'\varepsilon_{i,t-1}\varepsilon_{j,t-1}' a_j + f_i'\eta_{i,t-1}\eta_{j,t-1}' f_j$$

where $\eta_{t-1} \equiv (\eta_{1,t-1},\ldots,\eta_{N,t-1})'$ and $\eta_{i,t-1}\equiv\min(\varepsilon_{i,t-1},0)$.

The asymmetric dynamic covariance matrix model nests some natural extensions of the four models that allows for asymmetry in variances and covariances.

To check the performance of this model, we apply it to our large and small firm return series. The estimation results are reported in Table 3.[4] Notice that since ρ is significantly different from zero, the model cannot be simplified into the VECH, FAC or BEKK forms. Also, since ϕ and most of the covariance parameters are nonzero, the model cannot reduce to the CCOR model.

To check for misspecification, we apply the set of robust conditional moment tests to the model. With one exception, all the conditional moment tests, for both the variances and covariances, are insignificant. The exception is the test associated with the sign of $\varepsilon_{2,t-1}$ (the large firm residual) in the covariance equation, which is significant at the 2% level. Overall, we find minimal evidence of misspecification for the model.

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[4] In this model, we replaced the term $b_i' H_{t-1} b_j$ with $b_{ij} h_{ij,t-1}$.

Table 3: ADC Model Estimates

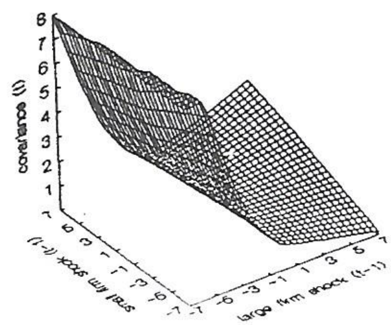
	estimate	standard error
ω_{11}	0.218	0.040
ω_{12}	–0.595	0.436
ω_{22}	0.027	0.019
a_{11}	0.217	0.015
a_{12}	–0.083	0.025
a_{21}	–0.070	0.057
a_{22}	0.254	0.033
f_{11}	0.075	0.041
f_{12}	–0.008	0.037
f_{21}	0.436	0.044
f_{22}	0.373	0.048
ρ_{12}	0.381	0.151
ϕ_{12}	0.626	0.163
β_{11}	0.868	0.014
β_{12}	0.495	0.241
β_{22}	0.884	0.015

Figure 2 plots the news impact surfaces implied by the ADC model. Interestingly, panel A shows that the covariance between large and small firm returns is higher following a negative shock to the large firm portfolio, while it is almost unaffected by shocks to the small firm portfolio. Next, panel B shows that the variance of the large firm portfolio is unaffected by small firm shocks, whether these shocks represent good news or bad news. This confirms the results of Conrad, Gultekin and Kaul (1991) who concluded that small firm news does not affect large firm volatility. On the other hand, volatility of the large firm portfolio increases after any news to the large firm, but especially after bad news. This could simply be a leverage effect. Finally, panel C indicates that the small firm portfolio has a dominant impact on small firm variances. This supports the findings of Conrad, Gultekin and Kaul (1991), who also show that large firm news spills over to small firm volatility. But it provides the additional insight that it is only the bad news that spills over, and not the good news.

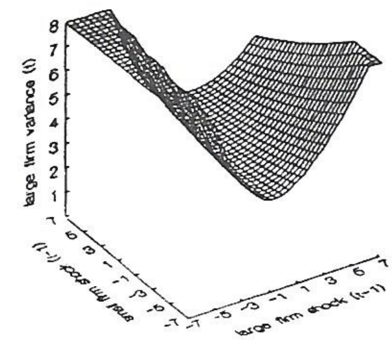
5. CONCLUSION AND SUMMARY

Several different multivariate GARCH models have been used in financial modeling. Each of these models has different implications about how past news impacts future variances and covariances. Caution must therefore be exercised when selecting a multivariate volatility model. We propose a set of misspecification indicators which can aid in this decision. We also show that the existing models misspecify the covariances of large firm and small firm equity portfolios. We therefore propose a more general model that nests most of the existing models, which passes most of the specification tests. Our model reveals that bad news to the large firm portfolio spills over to the small firm portfolio, but no other volatility spillovers exist.

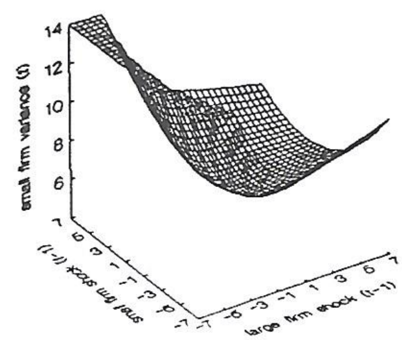
Figure 2: News Impulse Response Surfaces for the ADC Model



Covariance



Large firm variance



Small firm variance

6. REFERENCES

Amin, K. and V. K. Ng (1994), "A Comparison of Predictable Volatility Models using Option Data," unpublished manuscript, IMF Research Department.

Baillie, R. T. and R. J. Myers (1991), "Modeling Commodity Price Distributions and Estimating the Optimal Futures Hedge," *Journal of Applied Econometrics* 6, 109-124.

Black, Fisher (1976), "Studies in Stock Price Volatility Changes," *Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section, American Statistical Association*, 177-181.

Bodurtha, J. and N. Mark (1991), "Testing the CAPM with Time Varying Risks and Returns," *Journal of Finance* 46, 1485-1505.

Bollerslev, T. (1990), "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach," *Review of Economics and Statistics* 72, 498-505.

Bollerslev, T., R. Engle and J. Wooldridge (1988), "A Capital Asset Pricing Model with Time Varying Covariances," *Journal of Political Economy* 96, 116-131.

Bollerslev, T., R. Chou and K. Kroner (1992), "ARCH Modeling in Finance: A Selective Review of the Theory and Empirical Evidence," *Journal of Econometrics* 52, 5-59.

Chan, K., K.C. Chan and G.A. Karolyi (1991), "Intraday Volatility in the Stock Index and Stock Index Futures Markets," *Review of Financial Studies* 4, 657-683.

Chan, K.C., G.A. Karolyi and R. Stulz (1992), "Global Financial Markets and the Risk Premium on U.S. Equity," *Journal of Financial Economics* 32, 137-167.

Conrad, J., M. Gultekin and G. Kaul (1991), "Asymmetric Predictability of Conditional Variances," *Review of Financial Studies* 4, 597-622.

Engle, R. (1987), "Multivariate GARCH with Factor Structures – Cointegration in Variance,".unpublished manuscript, UC San Diego.

Engle, R.F. and K.F. Kroner (1995), "Multivariate Simultaneous Generalized ARCH," *Econometric Theory* 11, 122-150.

Engle, R. and V. Ng (1993), "Measuring and Testing the Impact of News on Volatility," *Journal of Finance* 48, 1749-1778.

Engle, R., V. Ng and M. Rothschild (1990), "Asset Pricing with a Factor ARCH Covariance Structure: Empirical Estimates for Treasury Bills," *Journal of Econometrics* 45, 213-238.

French, K., G.W. Schwert, and R. Stambaugh (1987), "Expected Stock Returns and Volatility," *Journal of Financial Economics* 19, 3-29.

Giovannini, A. and P. Jorion (1989), "The Time Variation of Risk and Return in the Foreign Exchange and Stock Market," *Journal of Finance* 44, 307-325.

Glosten, L., R. Jagannathan and D. Runkle (1993), "Relationship Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," *Journal of Finance* 48, 1779-1801.

Hamao, Y., R. Masulis and V. Ng (1990), "Correlations in Price Changes and Volatility Across International Stock Markets," *Review of Financial Studies* 3, 281-307.

Kroner, K. F. and S. Claessens (1991), "Optimal Dynamic Hedging Portfolios and the Currency Composition of External Debt," *Journal of International Money and Finance*, 10, 131-148.

Kroner, K. F. and V. K. Ng (1995), "Modeling the Time-Varying Comovement of Asset Returns," unpublished manuscript, University of Arizona.

Kroner, K. F. and J. Sultan (1993), "Time Varying Distributions and Dynamic Hedging with Foreign Currency Futures," *Journal of Financial and Quantitative Analysis* 28, 535-551.

Kuwahara, H. and T. Marsh (1992), "The Pricing of Japanese Equity Warrants," *Management Science*.

Nelson, D. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica* 59, 347-370.

Ng, L. (1991), "Tests of the CAPM with Time Varying Covariances: A Multivariate GARCH Approach," *Journal of Finance* 46, 1507-1521.

Ng, V., Engle, R., and M. Rothschild (1992), "A Multi Dynamic Factor Model for Stock Returns," *Journal of Econometrics* 52, 245-266.

Pagan, A. and G.W. Schwert (1990), "Alternative Models for Conditional Stock Volatility," *Journal of Econometrics* 45, 267-290.

Schwert, G.W. and P. Seguin (1990), "Heteroskedasticity in Stock Returns," *Journal of Finance* 45, 1129-1155.

Zakoian, J. (1994), "Threshold Heteroskedasticity Model," *Journal of Economic Dynamics and Control* 18, 931-955.