Journal of International Money and Finance (1993), **12**, 298–318

The impact of exchange rate volatility on international trade: reduced form estimates using the GARCH-in-mean model*

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In this paper, we use a multivariate GARCH-in-mean model of the reduced form of multilateral exports to examine the relationship between nominal exchange rate volatility and export flows and prices. The model imposes rationality on perceived exchange rate volatility, unlike conventional, two-step strategies. Tests are performed for five industrialized countries over the post-Bretton Woods era. We find that the GARCH conditional variance has a statistically significant impact on the reduced form equations for all countries. For most of the countries, the magnitude of the effect is stronger for export prices than quantities. In addition, the estimated magnitude of the impact of volatility on exports is *not* robust to using the conventional estimation strategy. (JEL F41, F31).

The perceived benefits of a flexible exchange rate system are insulation of the domestic economy from foreign shocks, and the potential for independent policy actions. However, increased uncertainty from high volatility in exchange rates can affect international trade, and thus might reduce the advantages of world-wide specialization. Decisions regarding exchange rate regimes and other exchange rate policies depend on optimally weighing these benefits and costs. It is therefore important to know if trade is influenced by exchange rate volatility.

Many empirical studies have examined this linkage using time series data, with mixed results (Cushman, 1988, pp. 317–318).[1] One potential problem with this

* The authors appreciate helpful comments from Devajyoti Ghose, Robert F. Engle, Adrian Pagan, Christopher G. Lamoureux, William Veloce, and Keivan Deravi. Lastrapes acknowledges financial support from the LSU College of Business and the University of Georgia Research Foundation.

0261-5606/93/02/298-21

literature is the *ad hoc* nature in which exchange rate volatility is measured. Typically, a simple function of past exchange rates, for example, a moving standard deviation (of arbitrary order) of past growth rates, serves as a proxy for time-varying volatility. This approach has the potential of ignoring important information on the stochastic process generating exchange rates. In addition, it requires a two-step estimation procedure (the first to estimate the volatility measure and the second to estimate the relationship), which may lead to inefficient estimators.

The purpose of this paper is to determine the reduced form impact of exchange rate volatility on international trade quantities and prices using a joint estimation technique in the context of a parameterized model of conditional variance. The empirical model allows joint estimation of the relationship between volatility and trade and how past information is related to perceived volatility. In effect, the model imposes rationality on the variance forecasts of market participants. This is accomplished by estimating a multivariate generalized autoregressive conditional heteroskedastic (GARCH) model, in which the variance of exchange rates appears in the conditional mean specification—GARCH-in-mean.[2] This model restricts the variance that affects trade to be the same as that generated by the data. Thus, our approach avoids the arbitrariness of the conventional tests by using the data to specify the variance forecast model. Tests are performed for five industrialized countries using monthly data.

The following section describes in detail the general empirical model, estimation techniques and the primary hypothesis tests. Section II discusses data and specification issues. In particular, we are careful to fully test and account for the stationarity properties of the data, including the estimated variances. Section III reports the estimates of the model, and compares these results to those using the naive measure of variance. A summary and conclusions follow in Section IV.

I. Empirical model

In this paper, estimates of the impact of exchange rate volatility on international trade are based upon the following model:

$$\langle 1 \rangle \qquad \Delta x_t = a_0 + a_1\,\Delta s_t + a_2\,\Delta p_t^* + a_3\,\Delta c_t + a_4\,\Delta y_t^* + d_x f(h_{t+1}) + a_5\,\Delta x_{t-1} + a_6\,\Delta x_{t-2} + a_7\,\Delta x_{t-3} + a_8 \varepsilon_{xt-1} + \varepsilon_{xt},$$

$$\langle 2 \rangle \qquad \Delta q_t = b_0 + b_1\,\Delta s_t + b_2\,\Delta p_t^* + b_3\,\Delta c_t + b_4\,\Delta y_t^* + d_q f(h_{t+1}) + b_5\,\Delta q_{t-1} + b_6\,\Delta q_{t-2} + b_7\,\Delta q_{t-3} + b_8 \varepsilon_{qt-1} + \varepsilon_{qt},$$

$$\langle 3 \rangle \qquad \Delta s_t = c_0 + \varepsilon_{st}.$$

x_t denotes real exports from the domestic country to the rest of the world during time period t, q_t is the corresponding price of exports denominated in foreign currency, and s_t is the foreign currency price of domestic currency. The right-hand-side variables include the ratio of foreign to domestic prices (p_t^*), domestic real unit labor costs (c_t), real foreign income (y_t^*) and a function of the time-varying conditional variance of the future exchange rate (h_{t+1}). Δ is the first difference operator. The εs are assumed to be white noise stochastic processes.

We impose a GARCH structure on the covariance matrix of the residuals in ⟨1⟩, ⟨2⟩, and ⟨3⟩. Define $\varepsilon_t = [\varepsilon_{xt}\, \varepsilon_{qt}\, \varepsilon_{st}]'$. Then,

$$\varepsilon_t | \varepsilon_{t-1}, \ldots \sim N(0, H_t),$$

⟨4⟩
$$H_t = \begin{pmatrix} \sigma_{xt}^2 & \sigma_{xqt} & 0 \\ \sigma_{xqt} & \sigma_{qt}^2 & 0 \\ 0 & 0 & h_t \end{pmatrix},$$

$$\sigma_{xt}^2 = \alpha_0 + \alpha_1 \varepsilon_{xt-1}^2 + \alpha_2 \sigma_{xt-1}^2,$$

$$\sigma_{qt}^2 = \beta_0 + \beta_1 \varepsilon_{qt-1}^2 + \beta_2 \sigma_{qt-1}^2,$$

$$h_t = \gamma_0 + \gamma_1 \varepsilon_{st-1}^2 + \gamma_2 h_{t-1},$$

$$\sigma_{xqt} = \delta_0 + \delta_1 \varepsilon_{xt-1} \varepsilon_{qt-1} + \delta_2 \sigma_{xqt-1}.$$

In general, the GARCH model assumes stochastic dependence between the current realization of $\varepsilon_t \varepsilon_t'$ and its past realizations. Thus, conditional variances and covariances are time-varying. The presence of h_{t+1} in the conditional mean equations implies that ⟨1⟩ through ⟨4⟩ is a multivariate GARCH-in-mean model.[3]

The model can be interpreted as the reduced form of a structural model of changes in real exports and export price. We do not attempt to identify structural (*e.g.*, supply and demand) coefficients from the reduced form, but focus on the determinants of exports and prices in equilibrium.

We assume that the contemporaneous right-hand-side variables are weakly exogenous with respect to the parameters in equations ⟨1⟩ and ⟨2⟩. Note in particular that s_t is assumed to be set independently of equilibrium in the market for exports; this is implied by the zero restrictions in H_t. The exchange rate and price level ratio are included to account for relative price effects on the supply and demand for exports. Unit labor costs are likely to affect domestic supply of exports, while foreign income can influence foreign demand for exports. These exogeneity assumptions and the inclusion of these explanatory variables are standard practice in the empirical trade literature.

To account for short-run conditional mean dynamics, we include an ARMA(3, 1) component in the trade and price equations. These terms can pick up serial correlation in the reduced form errors due, for example to lagged adjustment to changes in the exogenous variables, which has been shown to be important in the empirical trade literature. Because the nature of sluggish adjustment is not of primary interest in this paper, it is not necessary to specify lags of the exogenous variables explicitly. It is important to control for time dependence in the mean to avoid confusing these effects with the dependence implicit in the GARCH specification.

The exchange rate is specified as a martingale process, which is consistent with the well-known results of Meese and Rogoff (1983), as well as more recent evidence in Meese and Rose (1990) and Diebold and Nason (1990). This assumption implies that ε_{st} is unpredictable given observations on the past exchange rate, so that h_t measures the volatility of unexpected changes in s_t. GARCH models have been particularly successful in describing the properties of high-frequency exchange rate time series (*e.g.*, Hsieh, 1988; Lastrapes, 1989; and Baillie and Bollerslev, 1989). Note also that, given the statistical nature of our paper, we do not restrict the conditional variances of export quantity and price to be constant, but allow the data to determine the presence of time-varying volatility.

The primary objective of this paper is to test the significance and importance of fluctuations over time in exchange rate volatility as a determinant of exports and export prices. Such a relationship is implied by conventional theories of the effects of risk on international trade; these models (*e.g.*, Hooper and Kohlhagen, 1978) generally assume that risk averse traders respond to perceived exchange rate risk, which is proxied by the volatility of exchange rates. Although the typical presumption is that changes in exchange rate volatility negatively affect trade through this channel, de Grauwe (1988) has emphasized that the dominance of income effects over substitution effects can lead to a positive relationship between trade and volatility.

Exchange rate volatility can also influence export quantities and prices in hysteretic models of trade. When international transactions involve significant sunk costs, exchange rate uncertainty can affect trade behavior even when agents are risk neutral. In such a case, uncertainty can alter the option value of not participating in export activity (Baldwin and Krugman, 1989; Dixit, 1989). As with 'risk aversion' models, it is not always clear how trade will be affected. For example, Froot and Klemperer (1989, p. 643) show that exchange rate uncertainty can affect price and quantity of trade, either positively or negatively, when market share matters under an oligopolistic market structure, regardless of tastes for risk.

Because we do not identify a structural model, we are unable in this paper to isolate the cause of a relationship between trade and exchange rate volatility. However, our objective in estimating reduced form relationships is consistent with most of the trade/volatility literature.

The reduced forms therefore include a function of the conditional variance of the one-step-ahead exchange rate as a potential explanatory variable for export quantity and price. From alternative functional forms, which can account for potential non-linearities in the relationship, we choose the $f(h_{t+1})$ that yields the best fit to the data. The alternatives were h, $\ln h$, h^2, and $h_t^{\frac{1}{2}}$. The implicit assumption that the one-step horizon for the conditional variance is relevant for decision-making is not as restrictive as it seems—the GARCH specification allows for shocks to variance to be persistent. Thus, assuming the GARCH model is valid, the one-step variance is a good proxy for longer horizons. It is also assumed that volatility in nominal, rather than real, exchange rates is the relevant measure. However, under the current flexible rate regime, fluctuations in nominal and real exchange rates have been highly correlated over monthly horizons (Hakkio, 1989; and Mark, 1990). Because our empirical work focuses on the post-Bretton Woods era, the results are not likely to be sensitive to the use of real volatility.

As noted in the introduction, the GARCH-in-mean model imposes rationality on export market participants—the actual process that generates conditional variance determines economic behavior. Agents are assumed to know the parameters of this process, which is consistent with the empirical implementation of rational expectation models. This assumption is in contrast to the use of an *ad hoc* proxy for time-varying volatility. For example, taking a moving average of squared deviations from mean essentially sets γ_1 and γ_2 in $\langle 4 \rangle$ arbitrarily, without exploiting full knowledge of the process. While this conventional strategy approximates our specification, the full information approach that we use has the potential to generate more accurate and precise estimates of the relationship between volatility and trade.[4]

The parameters of the model in equations $\langle 1 \rangle$ through $\langle 4 \rangle$ are jointly estimated by maximizing the sum of the conditional log likelihood functions over the sample observations:

$$\ln L = \sum_{t=1}^{T} \ln L_t,$$

$$\langle 5 \rangle \qquad \ln L_t = \tfrac{1}{2}\ln(2\pi) - \tfrac{1}{2}\ln|H_t| - \tfrac{1}{2}\varepsilon_t' H_t^{-1}\varepsilon_t,$$

where T is the sample size. The algorithm of Berndt *et al.* (1974) is used to obtain estimates. Given initial values, this method updates the parameter estimates according to

$$\phi^{j+1} = \phi^j + \tau_j (S'S)^{-1} S \cdot i,$$

where ϕ is the vector of parameters, S is the matrix of scores, j is the iteration, τ_j is the step-size of iteration j, and i is a vector of ones. Numerical derivatives are used in the algorithm. $(S'S)^{-1}$ is an estimate of the covariance matrix of the estimators, which are FIML if the algorithm converges to a global optimum.[5]

II. Data and specification

We estimate the model using multilateral exports and the exchange rate adjusted unit value of exports, for five industrialized countries over the current floating rate period: the United States (US), the United Kingdom (UK), West Germany (WG), Japan (JP), and France (FR).[6] The data are monthly and range from 1973:1 for all countries to the latest date available for which the relevant data are reported: 1990:12 for the US and JP, 1990:11 for the UK and WG, and 1989:4 for FR. Given differencing and lags of the dependent variables, this data range allows the model to be estimated over a sample of 211 observations for the US and JP, 210 for the UK and WG, and 191 for FR.

Real exports (x) are obtained by deflating nominal exports by the appropriate unit value index. The nominal exchange rate (s) is the multilateral index based upon the multilateral exchange rate model of the IMF; this index is more relevant for total exports than a bilateral rate. To the extent that risk plays a role in explaining the relationship between volatility and the export market, the use of the multilateral data makes more plausible the implicit assumption that the risk is non-diversifiable. The foreign currency price of exports (q) is the unit value index deflated by the nominal exchange rate. Real unit labor costs (c) are proxied by the nominal unit labor cost index deflated by the consumer price index (CPI). The world price level (used to construct p^*) and world output (y^*) are trade-weighted averages of the CPI's and a proxy for real economic activity, respectively, from the 17 countries used in constructing the exchange rate—the weights are the same as those used in the exchange rate calculation. p^* is the ratio of the world price level to the domestic CPI. Additional details regarding the data, variable construction and sources are contained in Appendix B.

Preliminary tests suggested that real exports contain a monthly seasonal component. Therefore, this series was seasonally adjusted by subtracting the monthly mean from the raw series. This strategy turned out to be less burdensome on estimation than adding monthly seasonal dummies to the system.

Valid inference using the GARCH-M model requires that the variables in the system be stationary. Furthermore, if differencing is needed to induce stationarity, the correct specification of the model depends upon whether or not the variables are cointegrated. Thus, we perform tests for unit roots and cointegration as a means to identify the appropriate data transformation and model specification.

Unit root tests for the variables directly observable are straightforward. For all but one of the variables, x, q, s, p^*, c, and y^*, the null hypothesis of a single unit root in the univariate representation cannot be rejected at the 99 per cent critical value using the augmented Dickey and Fuller (1979) test with constant, trend, and four lags, while the null is rejected for the differences of each variable. This inference is generally confirmed by the Phillips and Perron (1988) test with truncated lag of six. Thus, first differencing these variables is necessary to eliminate the unit root in the mean representation of these variables.[7]

Testing for the stationarity of the latent variable $f(h_{t+1})$ raises some complex issues.[8] It is tempting to estimate the model and test for unit roots in the exchange rate conditional variance process (*i.e.*, integrated GARCH) along the lines suggested by Bollerslev and Engle (1990). However, the presence of the conditional variance in the mean makes this exercise misleading for several reasons. First, it is not clear what an integrated h process implies about the properties of a function of h, such as h^2. Second, and more importantly, the model that generates h_{t+1} depends upon the order of integration of h_{t+1}. For example, assume that $f(h_{t+1}) = h_{t+1}^2$ and that h_{t+1}^2 is stationary (*i.e.*, it is I(0)). Then h_{t+1}^2 is the appropriate variance term to include in $\langle 1 \rangle$ and $\langle 2 \rangle$. However, if h_{t+1}^2 has a unit root, then Δh_{t+1}^2, the stationary transformation, should be the form of the variance-in-mean variable. If the incorrect form is used in the mean, then the estimate of h_{t+1}^2 is inconsistent and unit root tests based on this conditional variance will be invalid. Finally, in the latter case, the model that generates h_{t+1} may be misspecified if the variables are cointegrated; in particular, an error correction term should be included. But this term cannot be determined unless h_{t+1} and its properties are known.

These inherent problems of the GARCH-in-mean model have not been addressed in the literature. We therefore propose the following approximate test for unit roots and cointegration. First, assume a particular functional form for $f(h_{t+1})$, say h_{t+1}^2. Second, estimate the model $\langle 1 \rangle$ through $\langle 4 \rangle$, except include h_{t+1}^2 and Δh_{t+1}^2 in the mean equations. This comprehensive specification permits consistent estimation of h_{t+1}^2 regardless of whether h_{t+1}^2 is I(0) or I(1). To understand why, suppose h_{t+1}^2 is I(0). By including both forms in the mean, we have

$$\lambda_1 \, \Delta h_{t+1}^2 + \lambda_2 h_{t+1}^2 = (\lambda_1 + \lambda_2) h_{t+1}^2 - \lambda_2 h_t^2,$$

where λ_1 and λ_2 are coefficients in the comprehensive specification. But since both h_{t+1}^2 and h_t^2 are I(0), the estimates of the conditional mean parameters are consistent, as are the estimates of the parameters in the conditional variance equation. Now, when h_{t+1}^2 is integrated, the estimator for λ_2 will converge to zero asymptotically (since its order of integration is different from the dependent variable), and that of λ_1 will converge to its true value. Asymptotically, the model is correct, which implies that h_{t+1} is consistently estimated, and the low frequency properties of the variance are maintained. The third step of our test procedure is to apply the augmented Dickey–Fuller test to the consistent h_{t+1}^2 series generated in step 2. The final step, assuming that h_{t+1}^2 is found to be I(1) in the previous step, is to test for cointegration of the variables in each mean equation, including the generated h_{t+1}^2, using the methods of Engle and Granger (1987).

Because comparisons of the optimized likelihood functions suggested that $f(h_{t+1}) = h_{t+1}^2$ outperformed the other functional forms, the augmented Dickey–Fuller tests for the null of a unit root in the generated h_{t+1}^2 are reported. The statistics are: (US) -2.26, (FR) -3.77, (JP) -1.20, (UK) -5.84, and (WG) -7.43. The 99 per cent critical value is -3.96, but, given that the data are generated, the true critical value is probably higher. However, it seems safe to conclude that the null can be rejected at reasonable levels of significance for WG and UK.[9]

For the US, FR, and JP, the Engle–Granger test for cointegration is performed for the levels of the variables in each reduced form equation. The cointegrating regressions are set up with x_t and q_t as dependent variables, which are regressed on the levels of the explanatory variables (including h^2, but excluding p^* from the JP system). The augmented Dickey–Fuller test statistics (with four lags) based upon the residuals from these regressions are, for x and q respectively, (US) -3.41, -2.19; (FR) -3.79, -2.88; (JP) -2.87, -2.53. The 99 per cent critical value from Engle and Yoo (1987) for a five variable system is -5.02. The critical value for our tests is likely to be larger (in absolute value) for two reasons: first, we have six variables in each equation (except for JP), and second, one of our variables is generated (*i.e.*, it is not directly observable). We conclude that there is no evidence for cointegration for these three countries. There was also no cointegration for the WG and UK equations. For these countries, the tests did not include h^2 in the cointegrating regressions.

It is clear that these stationarity tests are not exact. In particular, the tests are performed using generated variables, and, if cointegration does exist, the generated GARCH variances only approximate the true variances. However, there is no guidance in the literature for exact methods; finding such solutions is well beyond the scope of this paper. In any case, we feel that our approximate tests provide a firm foundation for valid inference regarding the effect of exchange rate volatility on the trade variables.

The implications of these preliminary tests for model specifications are: (1) for the US and FR, all variables, including h^2, are I(1), so that the system is estimated in first differences; (2) for JP, all variables are first differenced except p^*; (3) for the UK and WG, all variables are first differenced except h^2, which is incorporated in level form; and (4) for no country are the variables cointegrated, so that no error-correction terms are required to account for long-run interactions. The different specification for h^2 affects the permanence of the influence of volatility on trade, as will be discussed below.

In specifying the final equations to be estimated, we have been careful to fully control for dynamics in the mean and the covariance structure. The diagnostic test used to check for unexplained time dependence is the Ljung–Box Q-statistic applied to the standardized residuals and standardized squared residuals from the estimated equations. The former tests for serial correlation in the mean, while the latter is for serial correlation in variance (*i.e.*, remaining GARCH effects). We also apply the test to the standardized cross product of the residuals in the x and q equations. The Q-statistic tests the null that there is no linear time dependence up to a particular order of autocorrelation. The tests are performed up to a maximum lag order of 20 to determine the nature of any remaining serial correlation.

The tests (not reported) indicate that the ARMA(3, 1) specification is sufficient to capture the serial dependence in the means of the dependent variables. Although there appears to be some remaining seasonality in the residuals, it is quantitatively small according to the autocorrelation function and is unlikely to affect our results. For the squared residuals, the results suggest that the GARCH(1, 1) specification is sufficient to account for time dependence in the conditional variance of Δs, for all countries. Also, for all countries, there is no evidence of GARCH in the export equations or the covariance equation between export and price, so we set $\alpha_1 = \alpha_2 = \delta_1 = \delta_2 = 0$. Finally, the diagnostic tests imply that $\beta_1 = \beta_2 = 0$ (no GARCH in export price) for WG, which we also impose in estimation.

III. Empirical results

The joint estimation results are reported in Tables 1 through 5, for each country. The tables include the estimated coefficients and asymptotic t-statistics for two-models, a restricted version of the general model in which the exchange rate variance-in-mean terms are set to zero (*i.e.*, $d_x = d_q = 0$), and the unrestricted model in which this assumption is relaxed. We also report the likelihood ratio statistic which tests the validity of this restriction. The statistic has a χ^2 distribution with two degrees of freedom if the restriction is true.

The tables indicate that, of the conventional explanatory variables, real foreign income and the nominal exchange rate have the most consistent effects on exports and export prices, across countries. For all countries, income has a positive, significant effect on real trade, and exchange rates significantly affect export price. Exchange rates are fully passed through to foreign prices, on average, for the US, UK, and possibly WG, but pass-through is less for FR and JP. The price level ratio and real labor costs are generally insignificant, although there are exceptions.

There is general evidence that the dependent variables exhibit serial correlation. In almost every case, the autoregressive coefficients are significant, as are the moving average parameters.[10] As discussed above, the Ljung–Box Q-statistics indicate that the ARMA specification we use is adequate to filter dynamics in the mean. Note also that, almost always, inference regarding all explanatory variables is robust to the inclusion of the GARCH-M term.

The GARCH(1, 1) model provides a good fit for the monthly exchange rate data for all countries in the sample. In all cases, γ_1 and γ_2 are statistically significant according to the asymptotic t-statistics. Shocks to variance are strongly persistent, especially for the US, FR, and JP; the sum $\gamma_1 + \gamma_2$ exceeds 0.98 for these countries in the restricted model. This result is consistent with the tests in the previous sections that found variance shocks are permanent for these countries.[11] Diagnostic tests on the standardized residuals for excess skewness and kurtosis find no evidence that the conditional distribution is not normal.

TABLE 1. Estimation results for the United States (US)—1973:5 to 1990:12; Δh^2 in mean.

Parameter	Restricted Coefficient	Restricted t-statistic	Unrestricted Coefficient	Unrestricted t-statistic
a_0	−0.0522	−0.94	−0.0398	−0.74
a_1	−0.1705	−3.66	−0.1362	−2.56
a_2	0.2819	0.96	0.1648	0.58
a_3	−0.6738	−1.13	−0.7739	−1.30
a_4	0.6022	3.01	0.7563	3.75
a_5	0.4274	5.51	0.3923	5.24
a_6	0.1869	2.24	0.1621	2.05
a_7	0.2104	3.12	0.1962	2.68
a_8	−0.9900	[a]	−0.9900	[a]
α_0	46.2537	9.59	43.1213	9.39
b_0	0.3715	3.73	0.3742	3.75
b_1	1.1023	26.18	1.0963	23.97
b_2	−0.2585	−0.94	−0.2974	−1.04
b_3	0.4171	2.24	0.4193	2.16
b_4	−0.0768	−0.73	−0.0749	−0.70
b_5	0.0233	0.63	0.0308	0.80
b_6	−0.0388	−1.14	−0.0265	−0.76
b_7	0.0178	0.58	0.0194	0.63
b_8	0.0280	0.31	0.0151	0.17
β_0	0.0665	0.75	0.0651	0.74
β_1	0.1370	1.60	0.1186	1.52
β_2	0.8158	7.18	0.8333	7.50
δ_0	0.1911	0.32	0.1538	0.27
c_0	−0.0635	−0.50	−0.0322	−0.23
γ_0	0.0527	0.70	0.0449	0.74
γ_1	0.1560	2.60	0.0722	2.54
γ_2	0.8422	15.83	0.9174	26.70
d_x			−0.0664	−2.04
d_q			−0.0100	−0.86
LR (p-value)	7.04	(0.0296)		

Notes: The columns report the coefficient estimates and asymptotic t-statistics for the restricted and unrestricted models. LR is the likelihood ratio statistic that tests this restriction. p-Value is the marginal significance level of LR using the χ^2_2 distribution.

[a] The MA coefficient is fixed at −0.99 to ensure convergence.

We also find that export prices exhibit GARCH for the US, UK, FR, and JP. To our knowledge, this result has not been documented before. Although explaining the presence of GARCH in export prices lies outside the scope of this paper, it would be interesting to examine the extent to which changes in the conditional variance of exchange rates are 'passed-through' to the variance of export prices. This could perhaps be directly tested by specifying σ^2_{qt} as a function of ε^2_{st-i}.

TABLE 2. Estimation results for the United Kingdom (UK)—1973:5 to 1990:11; h^2 in mean.

Parameter	Restricted Coefficient	Restricted t-statistic	Unrestricted Coefficient	Unrestricted t-statistic
a_0	0.0182	0.31	0.1324	1.60
a_1	−0.0654	−1.72	−0.0837	−2.29
a_2	−0.0120	−0.16	0.0226	0.36
a_3	−0.0647	−0.40	−0.1053	−0.71
a_4	0.2791	2.34	0.2707	2.18
a_5	−0.0500	−0.57	−0.0352	−0.43
a_6	0.0663	0.76	0.0665	0.79
a_7	0.2570	3.14	0.2317	2.92
a_8	−0.9123	−19.51	−0.9374	−22.56
α_0	6.1160	9.33	5.9056	9.59
b_0	0.6748	7.56	0.5119	4.66
b_1	1.0012	29.71	0.9772	29.06
b_2	−0.0859	−0.84	−0.0755	−0.78
b_3	0.0031	0.05	0.0104	0.18
b_4	−0.1316	−2.04	−0.1320	−2.10
b_5	−0.1058	−2.49	−0.0827	−1.81
b_6	0.0273	0.65	0.0470	1.07
b_7	−0.0415	−1.24	−0.0449	−1.26
b_8	0.3558	4.34	0.3373	3.61
β_0	0.1870	2.85	0.1381	2.09
β_1	0.6005	4.90	0.5632	4.41
β_2	0.3506	3.81	0.4107	4.10
δ_0	−0.0899	−0.59	−0.0934	−0.64
c_0	−0.1442	−1.20	−0.1120	−0.91
γ_0	0.8613	2.06	0.7931	2.13
γ_1	0.2893	2.62	0.2590	2.65
γ_2	0.4080	2.16	0.4586	2.74
d_x			−0.0107	−1.53
d_q			0.0183	1.65
LR (p-value)	17.46	(0.0002)		

Notes: See notes to Table 1.

The final three rows in the table report estimates of the impact of nominal exchange rate volatility on export quantity and price. The coefficient on volatility in the export equation is significantly different from zero at a 5 per cent significance level for the US, a 10 per cent level for WG, 13 per cent for UK, and 17 per cent for FR. For export price, d_q is significantly different from zero only for the UK and, marginally, for JP. According to the likelihood ratio tests, the null hypothesis that the trade variables are jointly independent of the GARCH measure of exchange rate variance is convincingly rejected for each country in the sample. This statistic provides a more powerful test than the asymptotic t-tests, since the latter may be biased downward by collinearity in the system. While we find a

TABLE 3. Estimation results for France (FR)—1973:5 to 1989:4; Δh^2 in mean.

Parameter	Restricted Coefficient	Restricted t-statistic	Unrestricted Coefficient	Unrestricted t-statistic
a_0	0.1627	1.86	0.1589	1.77
a_1	−0.1068	−1.51	−0.0445	−0.57
a_2	0.8189	2.12	0.7266	1.92
a_3	0.2532	0.68	0.2898	0.84
a_4	0.4916	3.50	0.4896	3.72
a_5	−0.0900	−0.91	−0.0939	−0.93
a_6	−0.0006	−0.01	0.0090	0.11
a_7	0.2307	2.96	0.2405	2.96
a_8	−0.8426	−13.26	−0.8545	−13.72
α_0	7.0933	7.80	6.6389	7.63
b_0	0.4043	5.06	0.3978	4.90
b_1	0.4134	8.47	0.4138	8.28
b_2	−0.2753	−1.14	−0.3036	−1.23
b_3	−0.0096	−0.08	−0.0126	−0.10
b_4	−0.0659	−0.98	−0.0654	−0.94
b_5	−0.2082	−1.89	−0.1968	−1.77
b_6	−0.0367	−0.52	−0.0395	−0.55
b_7	−0.1125	−1.45	−0.1185	−1.50
b_8	−0.0561	−0.38	−0.0677	−0.46
β_0	0.0686	0.99	0.0663	0.94
β_1	0.2214	2.26	0.2348	2.25
β_2	0.6724	4.01	0.6712	4.01
δ_0	0.1834	1.15	0.1549	0.98
c_0	−0.1379	−1.82	−0.1731	−2.04
γ_0	0.0043	0.23	0.1602	2.09
γ_1	0.0629	3.19	0.2134	2.43
γ_2	0.9289	44.84	0.6849	6.25
d_x			0.1520	1.37
d_q			0.0010	0.12
LR (p-value)	10.06	(0.0065)		

Notes: See notes to Table 1.

statistical relationship between volatility and the trade variable, the direction of the effect differs across countries. For example, only for the US and UK is the impact of exchange rate volatility on trade negative.

Although it is reasonable to infer that exchange rate volatility has a statistically important effect on the trade reduced forms, it is not clear from the tables what the magnitude of the effect is. To quantify this effect, Tables 6 and 7 report the dynamic elasticities of export quantity and price with respect to variance shocks over various forecast horizons. In particular, Table 6 contains the dynamic multiplier $(\partial x_{t+k-1}/\partial h_{t+1})(h_m/x_m)$ for $k = 1, \ldots, 24$, where x_m denotes the sample mean of x and h_m is the unconditional variance of s $(\gamma_0/[1 - \gamma_1 - \gamma_2])$, while

TABLE 4. Estimation results for Germany (WG)—1973:5 to 1990:11; h^2 in mean.

	Restricted		Unrestricted	
Parameter	Coefficient	t-statistic	Coefficient	t-statistic
a_0	0.1441	2.03	0.0179	0.27
a_1	−0.0460	−1.01	−0.0408	−1.20
a_2	−0.2657	−1.25	−0.2268	−2.59
a_3	−0.1524	−2.08	−0.1085	−2.02
a_4	0.2505	2.74	0.2774	4.84
a_5	−0.2178	−2.58	−0.2161	−3.07
a_6	0.0470	0.54	0.0688	0.87
a_7	0.2381	2.94	0.2545	3.61
a_8	−0.8678	−15.00	−0.9900	a
α_0	2.6275	10.05	2.3557	11.02
b_0	0.2569	2.12	0.2311	1.54
b_1	0.8856	12.02	0.8762	12.00
b_2	0.1558	0.60	0.2342	0.92
b_3	−0.5806	−1.24	−0.0557	−1.22
b_4	0.0101	0.10	0.0077	0.08
b_5	−0.0724	−0.67	−0.0658	−0.93
b_6	−0.0617	−1.02	−0.0637	−1.08
b_7	−0.0012	−0.02	0.0059	0.10
b_8	−0.1971	−1.38	−0.2164	−1.57
β_0	1.0968	13.33	1.0645	13.91
δ_0	0.1765	1.58	0.0981	0.91
c_0	0.2592	2.79	0.2653	2.76
γ_0	0.4073	1.32	0.4193	1.57
γ_1	0.0974	2.28	0.1012	2.34
γ_2	0.6058	2.54	0.5925	2.94
d_x			0.0527	1.66
d_q			−0.0023	−0.04
LR (p-value)	19.86	(0.0000)		

Notes: See notes to Table 1.
[a] See notes to Table 1.

Table 7 contains the analogous multiplier for price. Note that we consider the dynamic response of the *level* of x_{t+k} to a change in the variance (not squared) of the exchange rate. This response function is obtained by solving the difference equations implied by the unrestricted model (*i.e.*, inverting the AR component) for the level of the dependent variables, then converting to percentage changes evaluated at x_m and h_m.

Consider first the export equations. The largest initial impact of volatility on real exports occurs for the US: a 1 per cent change in h_{t+1} causes a contemporaneous decline in multilateral exports of 0.015 per cent. The smallest initial elasticity is 0.0015 per cent for JP.

TABLE 5. Estimation results for Japan (JP)—1973:5 to 1990:12; Δh^2 in mean.

	Restricted		Unrestricted	
Parameter	Coefficient	t-statistic	Coefficient	t-statistic
a_0	1.0880	1.18	1.1389	1.17
a_1	−0.2155	−1.96	−0.2004	−1.85
a_2	−0.5003	−0.60	−0.5379	−0.61
a_3	−0.8952	−1.50	−0.9427	−1.59
a_4	1.1366	2.03	1.1287	1.94
a_5	0.2931	2.83	0.2979	2.76
a_6	0.0406	0.46	0.0421	0.48
a_7	0.2456	3.16	0.2393	3.17
a_8	−0.8486	−10.23	−0.8467	−9.67
α_0	131.3800	8.80	130.9850	8.76
b_0	1.5370	1.15	1.7575	1.34
b_1	0.5887	15.31	0.5517	13.46
b_2	−1.0166	−0.84	−1.2432	−1.03
b_3	0.0740	0.67	0.0784	0.78
b_4	0.2227	1.40	0.2610	1.65
b_5	−0.6165	−8.18	−0.6387	−7.80
b_6	−0.0251	−0.38	0.0061	0.08
b_7	0.0065	0.13	0.0204	0.44
b_8	0.5353	7.11	0.5834	8.64
β_0	2.2382	4.51	2.1519	5.14
β_1	0.3619	2.90	0.3827	3.07
β_2	−0.0158	−0.14	−0.0311	−0.34
δ_0	−4.8314	−4.14	−5.0263	−4.39
c_0	0.3811	1.61	0.3783	1.63
γ_0	0.3051	1.55	0.3082	1.61
γ_1	0.1366	2.47	0.1365	2.55
γ_2	0.8453	12.82	0.8440	13.19
d_x			0.0008	0.08
d_q			0.0050	1.55
LR (p-value)	9.50	(0.0087)		

Notes: See notes to Table 1.

Our tests for unit roots have important implications for the persistence of these effects on the dependent variables. Because the equations for the differenced quantity and price variables contain the *difference* of h_{t+1}^2 for the US, FR, and JP, shocks to h_{t+1} (*i.e.*, changes in the level of h, not its difference) are restricted to have no impact on the levels of the dependent variable at an infinite horizon.[12] However, for the UK and WG, such changes in the level of variance can have permanent effects. This implication is reflected in Table 6. Given the AR coefficient estimates, the initial negative effect of a one-time change in h_{t+1} declines almost monotonically to zero for the US. This damping occurs for the FR and JP cases as well. However, the impact of volatility is estimated to be permanent for the

TABLE 6. Relative dynamic response of real exports to percentage change in exchange rate volatility.

Horizon	US	UK	FR	WG	JP
1	−0.01469	−0.00413	0.01142	0.00613	0.00150
2	−0.00576	−0.00398	−0.00107	0.00481	0.00045
3	−0.00464	−0.00426	0.00020	0.00552	0.00020
4	−0.00564	−0.00520	0.00272	0.00683	0.00044
5	−0.00410	−0.00515	−0.00051	0.00626	0.00025
6	−0.00343	−0.00528	0.00012	0.00666	0.00014
7	−0.00312	−0.00549	0.00064	0.00687	0.00016
8	−0.00258	−0.00548	−0.00018	0.00670	0.00011
9	−0.00219	−0.00552	0.00005	0.00685	0.00007
10	−0.00189	−0.00557	0.00015	0.00686	0.00006
11	−0.00160	−0.00557	−0.00006	0.00683	0.00005
12	−0.00137	−0.00558	0.00002	0.00688	0.00003
13	−0.00117	−0.00559	0.00003	0.00687	0.00003
14	−0.00099	−0.00559	−0.00002	0.00686	0.00002
15	−0.00085	−0.00560	0.00001	0.00687	0.00002
16	−0.00072	−0.00560	0.00001	0.00687	0.00001
17	−0.00062	−0.00560	0.00000	0.00687	0.00001
18	−0.00052	−0.00560	0.00000	0.00687	0.00001
19	−0.00045	−0.00560	0.00000	0.00687	0.00001
20	−0.00038	−0.00560	0.00000	0.00687	0.00000
21	−0.00032	−0.00560	0.00000	0.00687	0.00000
22	−0.00028	−0.00560	0.00000	0.00687	0.00000
23	−0.00024	−0.00560	0.00000	0.00687	0.00000
24	−0.00020	−0.00560	0.00000	0.00687	0.00000
∞	0.00000	−0.00560	0.00000	0.00687	0.00000

Notes: The values reported are the dynamic elasticities $(\partial x_{t+k}/\partial h_{t+1})(h_m/x_m)$, for $k = 1, \ldots, 24$, where h_m is the unconditional variance of the change in the exchange rate and x_m is the sample mean of real exports. The final row reports the response at an infinite horizon.

remaining countries: a 1 per cent change in variance decreases the level of UK exports by 0.0056 per cent in the long-run, and increases WG exports ultimately by 0.0069 per cent.

The dynamic price elasticities in Table 7 are larger than those for export quantity. For the US, a 1 per cent change in h_{t+1} leads to a negative 0.33 per cent response in export price. The initial impact on JP price is 2.27 per cent, much larger than for the other countries. These responses are temporary, while the effects on UK and WG prices are permanent.

The motivation for this paper is to use an efficient estimation technique which imposes a form of rationality on export market participants. It is possible that the *ad hoc* tests in the literature, in which a simplified proxy for exchange rate volatility is used in a two-step approach, provide a good approximation to our approach. To compare the estimates from the two approaches in a controlled setting, we have reestimated the export equation separately from the other equations, substituting a six-month rolling sample variance (squared) for the

TABLE 7. Relative dynamic response of export price to percentage change in exchange rate volatility.

Horizon	US	UK	FR	WG	JP
1	−0.33314	0.32326	0.00878	−0.00867	2.27299
2	−0.01026	0.29653	−0.00173	−0.00810	−1.45176
3	0.00851	0.31393	−0.00001	−0.00759	0.94111
4	−0.00593	0.29672	−0.00097	−0.00771	−0.56357
5	−0.00061	0.30016	0.00040	−0.00773	0.33608
6	0.00030	0.29829	−0.00004	−0.00772	−0.19889
7	−0.00009	0.29938	0.00011	−0.00772	0.11759
8	−0.00002	0.29905	−0.00007	−0.00772	−0.06946
9	0.00001	0.29921	0.00001	−0.00772	0.04102
10	0.00000	0.29913	−0.00001	−0.00772	−0.02423
11	0.00000	0.29916	0.00001	−0.00772	0.01431
12	0.00000	0.29915	0.00000	−0.00772	−0.00845
13	0.00000	0.29915	0.00000	−0.00772	0.00499
14	0.00000	0.29915	0.00000	−0.00772	−0.00295
15	0.00000	0.29915	0.00000	−0.00772	0.00174
16	0.00000	0.29915	0.00000	−0.00772	−0.00103
17	0.00000	0.29915	0.00000	−0.00772	0.00061
18	0.00000	0.29915	0.00000	−0.00772	−0.00036
19	0.00000	0.29915	0.00000	−0.00772	0.00021
20	0.00000	0.29915	0.00000	−0.00772	−0.00012
21	0.00000	0.29915	0.00000	−0.00772	0.00007
22	0.00000	0.29915	0.00000	−0.00772	−0.00004
23	0.00000	0.29915	0.00000	−0.00772	0.00003
24	0.00000	0.29915	0.00000	−0.00772	−0.00002
∞	0.00000	0.29915	0.00000	−0.00772	0.00000

Notes: See notes to Table 6.

GARCH measure of exchange rate variance.[13] The parameter estimates of d_x, with t-statistics in parentheses, are:

US:	−0.058	(−1.26)
UK:	−0.007	(−1.98)
FR:	0.121	(1.66)
WG:	0.006	(1.40)
JP:	−0.003	(−0.08)

The *ad hoc* approach generally picks up the same signs on d_x as the GARCH model, and yields higher t-statistics for the UK and FR. On the other hand, under the single equation approach, we would not reject the null hypothesis that there is no impact of volatility on US trade. Furthermore, and more importantly, the estimates are smaller in absolute value than those for the GARCH model across the sample of countries. Since the only difference in the estimates is due to the nature of the volatility proxy, it seems to matter which approach is used to gauge the magnitude of the effect of exchange rate volatility on trade.

IV. Conclusion

In this paper, we have used a multivariate GARCH-in-mean model of the reduced form of the multilateral export market to examine the relationship between nominal exchange rate volatility and export flows and prices. The tests are performed for five industrialized countries over the post-Bretton Woods era. We find that the GARCH conditional variance has a statistically significant impact on the reduced form equations for all countries. The magnitude of the effect is generally stronger for export prices than quantities. In addition, the estimated magnitude of the impact of volatility on exports is *not* robust to using the conventional estimation strategy. Imposing rationality on market participants in proxying for perceived exchange rate volatility, which is accomplished in our empirical framework, therefore seems to be important.

Although we have chosen to focus on estimating reduced form models, our results have implications for structural models of multilateral trade. Clearly, structural trade models that include a role for exchange rate volatility are required to explain the behavior of our sample of countries. Furthermore, it is possible that fundamentally different models will be required to explain the divergence in the results across countries. The coefficient estimates, and the magnitudes of the effects, differ widely across the countries in our sample. Most importantly, our preliminary tests for stationarity, coupled with the model estimates, suggest that volatility has permanent effects on the trade variables for some countries, but not for others.

As with the previous empirical literature on trade and volatility, it is difficult to draw policy or welfare conclusions from our results. First, the magnitude of the impact of exchange rate volatility seems to be absorbed mostly in the price of exports. Second, the direction of the impact on trade differs across countries. Most importantly, only to the extent that there is a deadweight loss from under- (or over-) utilization of comparative advantage will the effect of exchange rate volatility on international specialization impose a welfare cost. The nature of this deadweight loss must be determined when considering the optimal exchange rate regime, or macro policies that affect exchange rate volatility. Our framework is not rich enough to investigate this issue.

Appendix A

To prove that the information matrix in the GARCH-M model in the text is not block diagonal with respect to the exchange rate equations, consider the following simple model:

$$y_{1t} = \sigma^2 + \beta x_t + \varepsilon_{1t} \tag{A.1}$$

$$y_{2t} = \varepsilon_{2t} \tag{A.2}$$

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & \sigma^2 \end{pmatrix}\right]. \tag{A.3}$$

This is an ARCH(0)-M model, and, though less general than the estimated model, retains the important property that the variance from one equation shows up in the mean of another equation. The log likelihood function for this system is

$$\ln L(\beta, \sigma^2) = -n \ln(2\pi) - \frac{n}{2} \ln \sigma^2 - \frac{1}{2} \sum_{t=1}^{n} (y_{1t} - \sigma^2 - \beta x_t)^2$$

$$-\frac{1}{2} \sigma^{-2} \sum_{t=1}^{n} y_{2t}^2, \tag{A.4}$$

where n is the number of observations. The relevant derivatives are

$$\frac{\partial \ln L}{\partial \beta} = \Sigma(y_{1t} x_t - \sigma^2 x_t - \beta x_t^2), \tag{A.5}$$

$$\frac{\partial^2 \ln L}{\partial \beta \, \partial \sigma^2} = -\Sigma x_t. \tag{A.6}$$

Therefore, $-E(\partial^2 \ln L / \partial \beta \, \partial \sigma^2) \neq 0$, which implies that the information matrix is not block-diagonal between the parameters in the mean of y_{1t} and those in the variance of y_{2t}. By ignoring the non-zero diagonal, the two-step estimator is inefficient.

Appendix B

This appendix describes the raw data, sources and construction of variables used in the empirical tests. All data are monthly, seasonally unadjusted (except y^*) and range from 1973:1 to 1990:12 (US and JP), 1990:11 (UK and WG), and 1989:4 (FR).

1. s—Nominal exchange rate.
   Effective (multilateral) exchange rate based on the MERM of the IMF; index, 1980 = 1 (world currency/domestic currency).
   Source: *International Financial Statistics* (*IFS*), line AMX.
2. x—Real multilateral exports.
   Total nominal exports in domestic currency deflated by unit values of exports; constant domestic currency units.
   Source for exports: *IFS*, line 70 (units of domestic currency).
   Source for unit value of exports: *IFS*, line 74.
3. q—Export price denominated in units of world currency.
   Unit value of exports multiplied by s.
4. c—Unit labor costs.
   Nominal unit labor costs deflated by consumer price index.
   Source for unit labor costs: *IFS*, 1985 = 1, line 65ey (US), line 65..c (UK), line 65 (FR and JP). *OECD Main Economic Indicators*, 1985 = 1 (WG).
   Source for consumer price index: *IFS*, line 64.
5. y^*—World output level.
   Geometric weighted average of output from the 17 other countries in the multilateral exchange rate index. The weights are the same as in the exchange rate index. The countries and proxy used for output are as follows:
   Australia—Industrial production index (IP), sa.
   Austria—IP, sa.
   Belgium—IP, sa.
   Canada—IP, sa.
   Switzerland—retail sales.
   West Germany—IP, sa.
   Denmark—retail sales, sa.
   Spain—IP, sa.

Finland—IP, sa.
France—IP, sa.
UK—IP, sa.
Ireland—retail sales, sa.
Japan—IP, sa.
Netherlands—IP, sa.
Norway—IP, sa.
Sweden—IP, sa.
USA—IP, sa.
Italy—IP, sa.
Source, for output proxies: *OECD Main Economic Indicators*. Retail sales were used for three countries because of the unavailability of IP data.

6. p^*—Ratio of world prices to domestic prices.
World price level is geometric weighted average of the CPIs from the 17 other countries used in the multilateral exchange rate index. The weights are the same as in the exchange rate index. World price is deflated by domestic CPI.
Source for CPIs: *OECD Main Economic Indicators*.

Notes

1. A nonexhaustive list includes Abrams (1980), Akhtar and Hilton (1984), Bailey *et al.* (1986, 1987), Cushman (1983, 1988), de Grauwe (1988), Gotur (1985), Hooper and Kohlhagen (1978), Kenen and Rodrick (1986), Koray and Lastrapes (1989), Perée and Steinherr (1989), and Thursby and Thursby (1985, 1987).
2. The seminal work on GARCH models is Engle (1982) and Bollerslev (1986). The GARCH-in-mean extension is due to Engle *et al.* (1987), and multivariate GARCH models are discussed and developed in Engle and Kroner (1993).
3. More general multivariate GARCH specifications exist; see Engle and Kroner (1993). The restricted version used here is adequate to explain our data. We also discuss below that the GARCH(1, 1) specification is sufficient for our data to capture dynamics in conditional variance.
4. Most applications of the GARCH-in-mean model have involved tests for time-varying risk premia (*e.g.*, Engle *et al.*, 1987; Domowitz and Hakkio, 1985; Baillie and Bollerslev, 1990; Diebold and Pauly, 1988; Bollerslev *et al.*, 1988; French *et al.*, 1987; and McCurdy and Morgan, 1987. See also the survey article by Bollerslev *et al.* (1992). Backus and Gregory (1988) criticize the use of GARCH-in-mean for making inferences about risk premia, but their criticism does not apply to our application.
5. Given the assumption of the exogeneity of the nominal exchange rate, it might at first appear that a two-step approach—estimating $\langle 3 \rangle$ and the last equation in $\langle 4 \rangle$, then using the implied h_{t+1} in $\langle 1 \rangle$ and $\langle 2 \rangle$—is efficient. This is not the case. The presence of h_{t+1} in $\langle 1 \rangle$ and $\langle 2 \rangle$ implies that the information matrix of the system is not block diagonal; thus, joint estimation is more efficient than the two-step approach. Proof of this assertion is given in Appendix A.
6. Canada was among our initial set of countries, but was dropped from the analysis because we could not achieve convergence.
7. Because these tests are standard, the results are not reported but are available upon request. For the level of real exports from JP, the Phillips–Perron test statistic indicated no unit root at 99 per cent. However, there was no indication of over-differencing in the estimated model for this country, so we maintain the assumption of a unit root. The only variable for which the level has no unit root is p^* for JP; therefore, its *level* is included in the estimated model for JP, but it is not incorporated in the cointegration tests.
8. Note that first-differencing the exchange rate eliminates the unit root in mean, but does not necessarily rule out other forms of nonstationarity, such as a unit root in the conditional variance. In our context, it is important to test for this latter form of nonstationarity in the exchange rate series since the conditional variance is an explanatory variable in the conditional mean equations.

9. It is comforting to note in Tables 1 through 5 below that only for these two countries is the sum $\gamma_1 + \gamma_2$ substantially less than one. Thus, our tests for unit roots in the generated variance are consistent with the notion of integration-in-variance.
10. For two countries, US and WG, the moving average coefficient exceeded one in absolute value, which led to non-invertibility and lack of convergence. We fixed this parameter to -0.99 which allowed the model to converge. Inference from the likelihood ratio test is unaffected by this restriction, since it is made for both models for the US and only the GARCH-M model for WG.
11. Technically, conventional tests for GARCH in the exchange rate equation breakdown for the GARCH-in-mean model. Under the null that the GARCH parameters γ_1 and γ_2 are zero, the conditional variance is constant and collinear with the constant in the trade equations. Thus, d_x and d_q are not identified under the null (see Davies, 1977). Because the GARCH coefficients are similar across specifications, and because this test is not the main focus of the paper, we make no adjustments to the test statistics.
12. For the countries with unit root h^2 processes, we have the following reduced form relation: $\Delta x_t = c(L)d_x \, \Delta h_{t+1}^2 + \cdots$, where $c(L)$ is the inverted AR lag polynomial. Thus, $\partial x_{t+k}/\partial h_{t+1}^2 = c_k d_x$, which converges to zero as k approaches infinity due to the stationarity of Δx. For the stationary h^2 processes, $\Delta x_t = c(L)d_x h_{t+1}^2 + \cdots$, or $x_t = (1 - L)^{-1}c(L)d_x h_{t+1}^2$. The lag polynomial on the right-hand side is the cumulated $c(L)$, which is not restricted to zero by the stationarity of Δx.
13. The order of the moving average is chosen to balance the competing objectives of capturing as much information as possible and maintaining a comparable sample size. However, unlike our approach, there is no other guidance in choosing this parameter.

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